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Is learning a dimension of risk?

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8 Abstract

9 We empirically assess how the uncertainty induced by investors' learning about the funda-
10 mentals affects stock returns. We identify two components of induced uncertainty: learning
11 uncertainty and dispersion of beliefs. We characterize these in terms of their relationship to
12 uncertainty about the fundamentals as estimated by surveys of economic forecasters and to
13 measures of uncertainty embedded in derivative markets (open interest and implied volatility).
14 We show that both learning uncertainty and dispersion of beliefs are conditionally priced.
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16 *JEL classification:* G11; G12; G14

17 *Keywords:* Conditional asset pricing; Time-varying risk factors; Learning uncertainty; Filtering

19 1. Introduction

20 The finance literature agrees on the existence of some “induced uncertainty”, gen-
21 erated by investors' processing of information. This uncertainty arises both from the
22 way in which investors update their beliefs about the fundamentals – *learning uncer-*
23 *tainty* – and from the way these beliefs differ across investors – *dispersion of beliefs*.
24 To date, little has been done to separate these two components and, in general, to

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25 empirically investigate the impact of induced uncertainty on stock returns. While it is
26 well known that the process of learning affects investors, generates uncertainty and
27 influences investor portfolio decisions, it is less well understood whether it aggregates
28 across investors in such a way as to affect returns, or whether it simply averages out.
29 One possible explanation of this fact lies in the standard framework hitherto used as
30 a basis for empirical tests: the Efficient Markets Hypothesis (EMH). EMH places
31 very strong restrictions on investors' learning. Investors start with arbitrary Gauss-
32 ian priors at the beginning of history and then they rationally update them by using a
33 Bayesian rule. All the history is assumed to be known and remembered.

34 These seemingly innocuous assumptions generate learning processes that quickly
35 converge to the long-run stationary equilibrium. Moreover, in the standard case of
36 normally distributed variables, this leads to a deterministic conditional variance.
37 However, learning uncertainty – i.e., the conditional variance – is the very dimension
38 of uncertainty forecasters use to gauge the reliability of their estimates and on which
39 portfolio managers assess the degree of riskiness of their strategies. It is also the key
40 variable used in business sensitivity analysis. Therefore, it is unfortunate that such a
41 variable turns out being a merely deterministic one, without a dynamics that can di-
42 rectly impact stock returns. To cope with this problem the literature has resorted to
43 two different approaches.

44 The first one is based on the relaxation of the assumption of Gaussian distributed
45 priors. This is enough to generate a stochastic conditional variance (Detemple,
46 1991). In this case, “the posterior beliefs of the investor are characterized by two sets
47 of sufficient statistics implementable in the form of a filter, (1) the vector of condi-
48 tional means and (2) a set of sufficient statistics for the conditional variance–covari-
49 ance matrix.” An alternative approach posits that investors forget and regularly
50 reassess their forecasts on the basis of a fresh set of priors and of a limited informa-
51 tion set based on a fixed window of observations. This approach has a flavor anal-
52 ogous to the recent models of bounded rationality due to limited memory
53 (Mullainathan, 1998). Individuals forget part of the history and use the same rati-
54 onal learning rule in the part they remember.¹ This approach is similar to the
55 one brought forward by Bossaerts (1999) that relaxes the assumption of predeter-
56 mined priors defined at the beginning of history and constantly updated. History
57 is truncated into “windows” and the time series dimension is collapsed in a series
58 of separate cross-sections. These take the form of (overlapping) histories character-
59 ized by “arbitrariness in the priors . . . at the starting point” of each of the histories.
60 While investors “may hold an arbitrary prior at the beginning of each history in the
61 dataset, in the subsequent history, however, they are required to use all the available
62 information to Bayesian update the priors, . . . as in the EMH”. That is, once the new
63 priors are determined at the beginning of each history, then investors behave as fully
64 rational agents. The advantage of this “near-rational” approach is that, while it

¹ In his model, the existence of forgetful individuals may lead to negative correlation between forecast errors over time.

65 “looses the natural connection [between the end of each period and the beginning of
66 another], gains in robustness to certain deviation of the EMH.”

67 Altering the learning window used by the investors to make forecasts provides a
68 link with behavioral theories. Indeed, as shown by [Brav and Heaton \(2002\)](#), the
69 behavioral bias of “representativeness” may be explained in terms of heavy weight-
70 ing of more recent data with respect to old one.

71 We rely on this approach and construct two proxies that separately identify learn-
72 ing uncertainty and dispersion of beliefs. In particular, we construct a trade-based
73 measure of uncertainty that mostly correlates with dispersion of beliefs and a
74 price-based measure of uncertainty that mostly correlates with learning uncertainty.
75 We then test whether this identification is correct, using outside measures of uncer-
76 tainty and dispersion of beliefs. These measures are macro-founded (survey of pro-
77 fessional forecasters) as well as micro-founded (dispersion of analysts’ forecasts,
78 open interest and implied volatility).

79 Once we have decomposed induced uncertainty into a part related to learning
80 uncertainty and another related to dispersion of beliefs, we examine whether they
81 are priced. To deal with the issue of time variation of the risk factors, we use a con-
82 ditional pricing model. We show that both learning uncertainty and dispersion of be-
83 liefs are conditionally priced. They represent components of the risk premium that
84 are separate from the fundamental one. We also find that learning uncertainty and
85 dispersion of beliefs affect the time-variation of the economic risk premiums.

86 The remainder of the paper is structured as follows. In Section 2, we present a
87 brief survey of the related literature. In Section 3, we lay out the hypotheses we test.
88 In Section 4, we describe how we construct our proxies for induced uncertainty and
89 how we identify them. We present the empirical results and report the main findings
90 in Section 5. A brief conclusion follows.

91 **2. Relation to existing literature**

92 There is a growing theoretical literature that focuses on the impact of learning on
93 optimal portfolio choice. For example, [Bawa et al. \(1979\)](#), [Kandel et al. \(1995\)](#), [Kan-
94 del and Stambaugh \(1996\)](#), and [Stambaugh \(1999\)](#) have thoroughly analyzed the ef-
95 fects of learning and estimation uncertainty on portfolio decisions. [Brennan \(1998\)](#)
96 shows how estimation risk, that is the risk of “learning bad news”, affects agents’
97 incentive to invest in stocks. [Barberis \(2000\)](#) argues that the process of learning
98 can tilt the portfolio composition so much that the need to hedge estimation risk
99 may prevail over the need to hedge systematic uncertainty. [Avramov \(2002\)](#) presents
100 evidence that model uncertainty is more important than estimation risk in portfolio
101 choice.

102 This literature has also extensively investigated the impact of learning on prices
103 ([Dothan and Feldman, 1986](#); [Gennotte, 1986](#); [Wang, 1993](#); [He and Wang, 1995](#);
104 [Brennan, 1998](#); [Barberis et al., 1998](#); [Veronesi, 2000](#); [Brennan and Xia, 2001b](#)).
105 [Xia \(2000\)](#) calibrates a single agent economy and finds significant effects produced

106 by agents' learning. Calibration to US data suggests that non-observability of ex-
107 pected dividend growth rate introduces an element of learning that increases the vol-
108 atility of stock prices (Brennan and Xia, 2001a). Lewellen and Shanken (2002) show
109 how parameter uncertainty is an important factor for characterizing and testing mar-
110 ket efficiency. Timmermann (1993) shows, in a partial equilibrium framework, that
111 systematic mistakes about future dividends are directly reflected in stock price
112 volatility.

113 However, the *empirical* issue of the relevance of these effects on asset prices has,
114 up to now, drawn little attention. Only recently, have Pastor and Veronesi (2003)
115 investigated the impact of learning uncertainty about stock profitability to explain
116 the market-to-book premium. This is all the more surprising as the empirical asset
117 pricing literature has extensively focused on identifying state variables that are priced
118 (Ferson and Harvey, 1991, 1993, 1997, 1999; Harvey, 1989, 1995; Bekaert and Har-
119 vey, 1995, 1997). Yet, none has tried to quantify the direct impact of learning on as-
120 set prices, assessing *whether it is actually priced, and how it relates to the time-*
121 *variation of risk premia.*

122 Even more problematic has been the empirical analysis of how dispersion of be-
123 liefs affects returns. While volume has been largely considered as a good proxy for
124 dispersion of beliefs and there is much convincing evidence of the *correlation* be-
125 tween trading volume and stock returns (Karpoff, 1987), *there is no evidence to sup-*
126 *port the claim that trading volume is a priced factor.* Indeed, trading volume is found
127 to be a “characteristic”, but not a risk factor (Chordia et al., 2001). Even the classic
128 identification of volume with dispersion of beliefs has been put in doubt. Diether et
129 al. (2002) and Ghysels and Juergens (2002) disregard volume and use the dispersion
130 of analysts' earning forecast as a proxy for dispersion of beliefs. While they docu-
131 ment the impact of dispersion of beliefs on stock returns, they do not relate it to vol-
132 ume, nor do they consider its time variation and its relation to learning uncertainty.

133 We take a different approach and rely on a “behavioral” extension of a standard
134 dynamic rational expectation models with asymmetric information (Wang, 1993; He
135 and Wang, 1995). We depart from rationality as investors, are “ignorant”, in the
136 sense that they apply the aforesaid sub-optimal learning rule, but they not aware
137 of their limitation (Brandt et al., 2001).² Therefore, they do not correct their mis-
138 takes. This is close to the idea of “rational structural uncertainty” (Brav and Heaton,
139 2002). Investors, even if aware of the limitation of the learning rule, do not incorpo-
140 rate it into their decision for the adjustment costs that this would entail. From a the-
141 oretical perspective, Simon (1955), Marschak (1968), Einhorn (1970, 1971), and
142 Payne et al. (1990) show how this behavior may be optimal if the computational
143 costs are high. From a more empirical perspective, the standard practice of exponen-
144 tial smoothing with fixed rolling windows used in commercial forecasting provides
145 evidence in this direction.

² Brandt et al. (2001) show that alternative learning behaviors alter significantly both the level and the time variation of the conditional moments of returns.

146 3. The hypotheses to be tested

147 We start with some notation. We define “learning uncertainty” the degree of
 148 uncertainty of investors’ forecasts. It is meant to capture the uncertainty that arises
 149 from the fact that investors learn about the fundamental value of the asset. Our
 150 proxy is the conditional variance of a filtering process that uses stock returns to infer
 151 the state of the economy. In particular, let us consider the following Kalman filter:

$$\begin{cases} X_{t+1} = AX_t + u_t, \\ \mathbf{R}_t = \mathbf{B}X_t + \mathbf{v}_t, \end{cases} \quad (1)$$

155 where \mathbf{R}_t is the vector of returns of the assets and X_t is the underlying state which the
 156 returns are a function of. We can think of X_t as the main factor affecting the econ-
 157 omy (e.g., GDP). \mathbf{R}_t is the vector of portfolio returns constructed, in the spirit of
 158 Fama and French (1993), aggregating the stocks in either 25 value-weighted portfo-
 159 lios on the basis of the book-to-market and size classification, or 30 portfolios on the
 160 basis of four-digit SIC codes. The residuals u_t and v_t are Gaussian white noise se-
 161 quences with $Var(u_t) = Q$ and $Var(\mathbf{v}_t) = \mathbf{V}$, respectively. Our measure of learning
 162 uncertainty is the investors’ *conditional variance* of the underlying state (i.e.,
 163 $LU_t = Var[X_{t+1} | \{\mathbf{R}\}_{1, \dots, t}]$).

164 A simple example will convey the intuition behind our proxy LU_t . Let us, for
 165 example, consider an investor who knows that stocks returns are a function of the
 166 state of the economy (i.e., GDP). He does not observe GDP and can only use the
 167 information contained in stock returns to infer it. Therefore, every morning, he ob-
 168 serves stock returns (\mathbf{R}_t) and uses them to determine GDP (X_t) and forecast its value
 169 in the future. The future value of GDP will allow him determine the correct stock
 170 returns and invest accordingly. The system of Eq. (1) is meant to reproduce such
 171 a forecasting process. That is, at each point in time, the investor looks backward
 172 a certain number of days and uses the past stock returns to train his filter. The con-
 173 ditional variance (LU_t) represents the uncertainty about his forecast. The higher the
 174 variance, the less accurate the forecast is and the more uncertain the investor will
 175 feel. This higher uncertainty will be reflected in his portfolio choice and, in equilib-
 176 rium, in the stock prices.

177 Our second measure of induced uncertainty tries to capture the dispersion of be-
 178 liefs among investors. We define “dispersion of beliefs” the differences in the ex-
 179 pected value that investors, characterized by different information sets, attribute to
 180 an asset or to the state variables that determine the value of the asset (e.g., future
 181 dividends).

182 We consider a rational expectation framework where investors are differentially
 183 informed. The fact that investors are not fully informed induces them to “learn”.
 184 This learning generates uncertainty against which investors want to hedge. That is,
 185 investors attempt to protect themselves against the risk of “learning bad news: that
 186 the mean (return) is low” (Brennan, 1997). Higher learning risk prompts the inves-
 187 tors to adjust their portfolios to hedge the increased risk. The fact that portfolio deci-
 188 sions are conditioned by this learning uncertainty implies that in equilibrium stock

189 prices are affected by it and that learning uncertainty becomes an additional risk
190 factor.

191 We make two assumptions that make us depart from the standard framework.
192 First, we assume that investors “re-evaluate” their assessment of the state of the
193 world by using a new set of priors.³ The updating is a standard Kalman updating
194 process. The second assumption is about limited memory. In the standard frame-
195 work, the whole history matters and the statistics are a function of it. We, on the
196 contrary, assume that investors update their estimates on the basis of a fixed rolling
197 window of observations. This is consistent with models of bounded rationality due to
198 limited memory (Mullainathan, 1998) as well as with models based on rational struc-
199 tural uncertainty (Brav and Heaton, 2002). We assume that investors “forget”. In
200 particular, at each time t , investors only consider the previous $t - N$ data points.
201 As in Bossaerts (1999), history is truncated into a series of “windows” and a new
202 set of priors is used at each point in time. These take the form of (overlapping) his-
203 tories characterized by “arbitrariness in the priors . . . at the starting point” of each
204 of the histories. The joint effect of different priors, reassessment of the assumed
205 underlying parameters and limited memory breaks the continuity of learning of
206 the investors and, effectively, makes the conditional variances stochastic.

207 We assume information asymmetry and a nested asymmetric information struc-
208 ture – i.e., the informed investors have access to the same information of the less in-
209 formed (i.e., prices) and to something else. This allows the more informed investors
210 to condition their trading decisions also on the forecasts of the less informed and im-
211 plies that the learning errors of the less informed become themselves additional “fac-
212 tors” that affect stock returns. We can summarize these two intuitions with the
213 following claim.

214 *Claim 1: If there is information asymmetry and information is nested, learning*
215 *uncertainty and dispersion of beliefs affect both stock returns and trading volume.*

216 This claim is based on an extension of Theorem 4.2, 4.3, and 4.4 of Wang (1993).
217 The test of the restriction that dispersion of beliefs is priced is, in fact, a joint test of
218 pricing and asymmetry of information.

219 The aforesaid discussion provides a direct way of constructing a proxy for learn-
220 ing uncertainty. Moreover, the dynamic nature of the relationship between trading

³ This may happen in the case of a change in the composition of the investors in the economy. Let us assume, for example, that at the end of each trading period a new cohort of investors enters the economy with different priors. We may think of the new cohort as a change in the overall composition of the investors in the economy as well as a rotation of the class of marginal investors. The new investors will re-elaborate the same information contained in the observed signals (i.e., prices) by using their own priors. This approach would suggest that the different cohorts of investors would use the same rational learning (i.e., they have the same likelihood), but adopt different priors. This will lead to new beliefs that will be impounded in prices. This interpretation would also be consistent with the “differential interpretation hypothesis”, that is with the idea that investors have different beliefs about the distribution of the signals (Kandel and Pearson, 1995; Kandel and Zilberfarb, 1999). If the new class of investors interprets the signals differently, their different beliefs will be impounded into prices.

Table 1
Descriptive statistics for stock portfolios

Size and book-to-market portfolios (25 portfolios)				Industry portfolios (30 portfolios)			
Portfolio	Mean (RET)	Median (RET)	Std. Dev. (RET)	Portfolio	Mean (RET)	Median (RET)	Std. Dev. (RET)
FF01 (BM1 SZ1)	-0.20	-0.01	8.07	Food	0.73	0.69	4.70
FF02 (BM2 SZ1)	0.49	0.77	7.01	Beer	0.75	0.67	5.84
FF03 (BM3 SZ1)	0.73	0.93	6.31	Smoke	1.06	1.08	5.77
FF04 (BM4 SZ1)	0.89	1.24	6.12	Games	0.78	0.92	7.23
FF05 (BM5 SZ1)	1.28	1.14	6.30	Books	0.65	0.67	5.76
FF06 (BM1 SZ2)	0.19	0.51	7.28	Cons Goods	0.59	0.70	5.01
FF07 (BM2 SZ2)	0.51	0.84	6.57	Apparel	0.66	0.43	7.56
FF08 (BM3 SZ2)	0.70	0.89	5.79	Health	0.76	0.73	5.28
FF09 (BM4 SZ2)	0.92	1.20	5.56	Chemicals	0.43	0.44	5.29
FF10 (BM5 SZ2)	1.16	1.49	5.85	Textiles	0.47	0.50	6.24
FF11 (BM1 SZ3)	0.24	0.48	6.45	Construction	0.50	0.30	5.80
FF12 (BM2 SZ3)	0.57	0.84	5.80	Steel	0.15	0.20	6.27
FF13 (BM3 SZ3)	0.59	0.86	5.23	Fabricated prod	0.31	0.52	5.91
FF14 (BM4 SZ3)	0.84	1.12	4.97	Electrical Eq	0.58	0.62	6.05
FF15 (BM5 SZ3)	1.02	1.25	5.85	Autos	0.42	0.10	5.87
FF16 (BM1 SZ4)	0.36	0.40	5.68	Carry	0.71	0.72	6.61
FF17 (BM2 SZ4)	0.47	0.66	5.34	Mines	0.30	0.08	7.26
FF18 (BM3 SZ4)	0.52	0.77	4.74	Coal	0.56	0.43	7.97
FF19 (BM4 SZ4)	0.81	1.08	4.96	Oil	0.54	0.60	5.19
FF20 (BM5 SZ4)	1.06	1.23	5.63	Util	0.42	0.37	3.90
FF21 (BM1 SZ5)	0.50	0.73	4.72	Telecom	0.54	0.55	4.31
FF22 (BM2 SZ5)	0.37	0.61	4.50	Services	0.78	0.86	6.74
FF23 (BM3 SZ5)	0.45	0.55	4.35	Business equipment	0.47	0.10	5.91
FF24 (BM4 SZ5)	0.63	0.87	4.48	Business supplies	0.45	0.49	5.17
FF25 (BM5 SZ5)	0.73	0.85	5.00	Transport	0.43	0.80	6.63
				Wholesale	0.77	0.79	6.53
				Retail	0.63	0.23	5.73
				Restaurants/hotels	0.73	1.07	6.95
				Finance	0.60	0.63	5.34
				Other	0.30	0.37	4.11

We report the descriptive statistics for the 25 size and book-to-market portfolios and for the 30 industry portfolios. The size and book-to-market portfolios are formed using the Fama and French (1993) procedure. The portfolios, which are constructed at the end of each June, are the intersections of five portfolios formed on size (market equity, ME) and five portfolios formed on the ratio of book equity to market equity (BE/ME). The size breakpoints for year t are the NYSE market equity quintiles at the end of June of t . BE/ME for June of year t is the book equity for the last fiscal year end in $t - 1$ divided by ME for December of $t - 1$. The BE/ME breakpoints are NYSE quintiles. Portfolios include NYSE and AMEX firms for 1964–1982 and NYSE, AMEX and NASDAQ firms for 1983–1998. To be included in the sample, the firm must have volume reported for given year. Descriptive statistics are compiled for June 1964–December 1998. We report statistics for the excess value-weighted returns RET (mean, median, and standard deviation, % per month).

221 volume, dispersion of beliefs and learning uncertainty suggests the type of pricing
222 model to use for our empirical investigation. Indeed, if dispersion of beliefs changes

223 over time, a *conditional pricing model* should better capture the covariation between
224 returns and trading volume. We now move on to the construction of our measure of
225 induced uncertainty.

226 4. Measuring induced uncertainty

227 Our goal is to construct measures of induced uncertainty that proxy for dispersion
228 of beliefs and learning uncertainty and to relate them to stock returns. The main
229 problem is the fact that the proxies that have been used in the literature are observed
230 at too low a frequency (e.g., macro-based measures). Alternatively, their availability
231 is limited in time (e.g., micro-based measures exploiting the information from the
232 derivatives market). We will therefore proceed in two steps. First, we use financial
233 data (i.e., returns and trading volume) to construct proxies for the two components
234 of induced uncertainty. Then, we exploit already existing measures to verify the qual-
235 ity of our proxies. For this purpose, we will use macro- and micro-based measures of
236 uncertainty.

237 4.1. A price-based measure of learning uncertainty

238 We construct LU_t by estimating the Kalman filter represented by the system of
239 Eq. (1). We concentrate on the minimum information set *common to all the investors*
240 *in the market*: observable stock returns. To determine the estimation window, we
241 consider monthly intervals. In other words, we reestimate the Kalman filter at the
242 end of each month on the basis of rolling windows made of the previous 500 trading
243 days. This implies that investors' LU_t s at the end of the month are based on the
244 observations of the previous 500 days, conditional on the new set of hypothesized
245 parameters. Each month a new 500-day update is determined based on a new set
246 of parameters. Time series of LU_t s are calculated for the period June 1964–Decem-
247 ber 1998. The size of the window implicitly defines the memory length of the
248 investors.⁴

249 While LU_t is meant to capture learning uncertainty, it may, in fact, be proxying
250 for stock market volatility. In order to ascertain this, we regress our measure of
251 learning uncertainty on the monthly estimates of volatility of returns based on daily
252 observations of CRSP indexes. We use both equally- and value-weighted indexes.
253 The results display a very low correlation between the two variables (*Adjusted R*²
254 is below 1.3%). This suggests that our *measure contains information different from*
255 *that embedded in the standard measures of volatility.*

⁴ Alternatively, we consider quarterly intervals. That is, we reestimate the Kalman filter at the end of each quarter, on the basis of *non-overlapping* rolling windows made of the previous three months' daily returns data. Given that the use of non-overlapping windows dramatically reduces the number of degrees of freedom, in this case, we use as input an aggregate index.

256 4.2. A trade-based measure of dispersion of beliefs

257 Our second measure of induced uncertainty tries to capture the dispersion of be-
258 liefs among investors. A possible proxy is the dispersion in analyst forecasts. How-
259 ever, these are available only from 1986. In the case of the standard deviation of
260 macro-forecasts, instead, while available for a longer period, they come at a lower
261 frequency (quarterly). We therefore consider a “trade-based” measure of uncer-
262 tainty, – i.e., is based on the aggregate market trading volume or turnover. Trading
263 volume is a direct function of both dispersion of beliefs and learning uncertainty.⁵
264 Therefore, trading volume, once purged of the effects of learning and fundamental
265 uncertainty, should be a good proxy for investors’ dispersion of beliefs.

266 We proceed as follows. We first define, for each individual stock, turnover as the
267 logarithm of the monthly ratio between total number of shares traded and the total
268 number of shares outstanding. This measure, similar to the one used by Lo and
269 Wang (2000) and Chordia et al. (2001), allows us to aggregate turnover across stocks
270 of companies that are different in terms of size and number of share outstanding.⁶

271 Given that the two measures of induced uncertainty are related, we orthogonalize
272 them by regressing turnover on LU_t and taking the residuals as our measure of
273 trade-based uncertainty ($DISP_t$). This captures the component of turnover that is
274 orthogonal to LU_t . The orthogonalization is done once for each sample we estimate
275 (i.e., full sample and different sub-samples). Table 2, Panel A and B, report the
276 descriptive statistics for the price-based measure, the trading-based measure and
277 the components of the two measures orthogonal to three Fama and French (from
278 here on “FF”) factors. Table 2, Panel C, reports the autocorrelation coefficients
279 up to 12th lag, Box–Ljung statistics and the results of Phillips–Perron unit root tests
280 (with trend). Both variables appear to be stationary.⁷ It is also interesting to note
281 that LU_t is hardly related to any of the FF factors, while $DISP_t$ is positively related
282 to HML.

⁵ That is, trading volume is driven by investors’ individual information sets and by their differences with respect to the common one. In other words, trading volume is a function of investors’ learning uncertainty based on both their *private and common information sets*, while prices are a function of the learning uncertainty based only on investors’ *common information set*.

⁶ We experimented with different aggregations across stocks: logarithm of average monthly turnovers of the stocks part of the portfolio, the average of the logarithm of the monthly turnover of each single stock and the average monthly turnover of the stocks in the portfolio. Given that the results in the three specifications agree, we report only the ones based on the logarithm of average monthly turnovers of individual stocks. The other specifications are available upon requests from the authors. Moreover, to account for the fact that the volume for NASDAQ-listed stocks reported in CRSP is overstated due to double counting of the transactions between NASDAQ market-makers, we multiply volume for NASDAQ stocks by 0.56 (Atkins and Dyl, 1997).

⁷ The tests of stationarity without trend and the augmented Dickey–Fuller test are available upon request from the authors. The results agree with those reported. It is interesting to note that on the level of individual firms, the measures of trade are often found to be non-stationary (Lo and Wang, 2000).

Table 2

Measures of induced uncertainty

Variable	Lu		DISP					
	Coefficient	<i>t</i> -Statistic	Coefficient	<i>t</i> -Statistic				
<i>Panel A: Induced uncertainty and FF factors</i>								
Intercept	1.28	(43.12)	−0.25	(−0.31)				
MKT_RF	−1.48	(−1.41)	4.91	(0.15)				
SMB	−0.87	(−0.70)	−13.05	(−0.37)				
HML	−0.53	(−0.69)	59.74	(2.56)				
Adj. R^2	0.001		0.024					
Variable	Mean	Median	Std. Dev.	Minimum	Maximum	Skewness	Kurtosis	
<i>Panel B: Descriptive statistics of induced uncertainty</i>								
LU	1.27	1.13	0.58	0.44	3.24	0.62	−0.41	
DISP	0.00	−1.44	16.11	−27.85	52.77	0.57	−0.13	
LU ⁰	0.00	−0.16	0.58	−0.88	1.98	0.63	−0.37	
DISP ⁰	0.00	−2.20	15.85	−34.55	55.64	0.62	0.05	
Lags	LU		DISP		LU ⁰		DISP ⁰	
<i>Panel C: Autocorrelation structure</i>								
1	0.22		0.88		0.22		0.86	
2	0.25		0.83		0.25		0.81	
3	0.12		0.79		0.11		0.78	
4	0.13		0.74		0.12		0.73	
5	0.16		0.74		0.16		0.74	
6	0.18		0.72		0.17		0.72	
7	0.22		0.70		0.21		0.71	
8	0.19		0.69		0.18		0.69	
9	0.07		0.68		0.08		0.68	
10	0.12		0.66		0.12		0.67	
11	0.10		0.65		0.10		0.64	
12	0.05		0.65		0.05		0.65	
<i>Box–Ljung statistics</i>								
$Q(24)$	141		3479		132		3429	
$p < \chi^2$	<0.0001		<0.0001		<0.0001		<0.0001	
<i>Phillips–Perron unit root test (with trend)</i>								
Z_ρ	−375		−72		−375		−90	
$p < Z_\rho$	<0.0001		<0.0001		<0.0001		<0.0001	

Panel A reports the results of the regressions of our measures of induced uncertainty (LU and DISP) on the Fama and French factors. We use the Newey–West estimator with correction for autocorrelation. The residuals are the orthogonalized factors (LU⁰ and DISP⁰ respectively). In Panel B, we report the descriptive statistics of the LU and DISP and their components orthogonal to factors (LU⁰ and DISP⁰). Panel C reports the autocorrelation coefficients up to 12th lag, the Box–Ljung statistics and the results of Phillips–Perron unit root test (with trend). The lags are months. All the estimations are done for the period 1964:06–1998:12.

Table 3

Induced uncertainty and surveys

Variable	Start Date	Mean	Std. Dev.	Min	Max	Median number of responses per survey	Correlation between 6- and 12-month variable
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Panel A: Standard deviation of macro-economic forecasts: Livingston Survey (descriptive statistics)

RTL6	1959	0.050	0.183	0.009	1.521	38	0.191
RTL12	1959	0.037	0.011	0.014	0.088	38	
AUTO6	1966	0.060	0.025	0.025	0.140	47	0.935
AUTO12	1966	0.071	0.028	0.029	0.165	47	
IP6	1946	0.018	0.006	0.007	0.030	52	0.488
IP12	1946	0.027	0.012	0.010	0.097	51	
GDPX6	1946	0.013	0.005	0.006	0.024	55	0.838
GDPX12	1946	0.019	0.006	0.009	0.030	54	
UNP6	1961	0.053	0.018	0.024	0.106	53	0.847
UNP12	1961	0.076	0.024	0.037	0.136	53	
WAGE6	1949	0.017	0.013	0.008	0.114	36	0.266
WAGE12	1949	0.023	0.007	0.012	0.051	36	
CPROF6	1971	0.082	0.037	0.025	0.161	46	0.898
CPROF12	1971	0.102	0.041	0.039	0.195	45	

Forecast	Mean	Std. Dev.	Min	Max	Median number of responses per survey	Correlations			
						2 Quarters ahead	3 Quarters ahead	4 Quarters ahead	5 Quarters ahead

*Panel B: Probability of recession: ASA-NBER Survey**Mean probability*

1 Quarter ahead	20.49	23.99	0.52	88.71	34	0.88	0.62	0.26	-0.17
2 Quarters ahead	20.34	17.21	2.16	74.06	34		0.87	0.50	-0.01
3 Quarters ahead	19.16	11.72	4.03	58.80	34			0.79	0.29
4 Quarters ahead	18.13	7.18	4.56	36.31	34				0.78

Standard deviation (σ_π)

1 Quarter ahead	19.30	13.24	2.47	55.25	34	0.73	0.44	0.21	0.05
2 Quarters ahead	19.60	13.24	1.19	56.44	34		0.75	0.42	0.19
3 Quarters ahead	18.81	13.61	0.93	56.76	34			0.71	0.45
4 Quarters ahead	18.09	13.81	0.72	68.26	34				0.81

Index of uncertainty $I_\pi = \pi(1 - \pi)$

1 Quarter ahead	0.106	0.073	0.001	0.250	34	0.82	0.53	0.20	-0.11
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(continued on next page)

Table 3 (continued)

Forecast	Mean	Std. Dev.	Min	Max	Median number of responses per survey	2 Quarters ahead	3 Quarters ahead	4 Quarters ahead	5 Quarters ahead
2 Quarters ahead	0.133	0.067	0.021	0.250	34		0.82	0.45	0.04
3 Quarters ahead	0.141	0.055	0.039	0.250	34			0.79	0.37
4 Quarters ahead	0.143	0.043	0.044	0.231	34				0.82

Variable	Model 1		Model 2		Model 3		Model 4	
	Estimate	<i>t</i> -stat.	Estimate	<i>t</i> -stat.	Estimate	<i>t</i> -stat.	Estimate	<i>t</i> -stat.
<i>Panel C: Induced uncertainty and dispersion of forecasts (the Livingston Survey)</i>								
<i>LU</i>								
Intercept	0.61	(2.02)	0.45	(1.51)	0.90	(3.25)	0.71	(2.17)
RTL6	0.49	(2.92)	0.44	(1.99)	0.56	(3.43)	0.51	(2.41)
RTL12	5.58	(0.95)	6.02	(1.02)	5.58	(0.95)	6.02	(1.02)
AUTO6	-23.08	(-3.86)	-27.19	(-4.82)	-0.41	(-0.14)	-2.56	(-0.76)
AUTO12	22.02	(3.78)	23.91	(4.24)	22.02	(3.78)	23.91	(4.24)
IP6	23.07	(1.56)	27.69	(1.66)	20.79	(1.58)	22.84	(1.58)
IP12	-2.27	(-0.55)	-4.83	(-1.08)	-2.26	(-0.55)	-4.82	(-1.08)
GDPX6	18.48	(0.65)	34.96	(1.17)	-12.58	(-0.95)	-12.12	(-0.60)
GDPX12	-29.78	(-1.01)	-45.16	(-1.05)	-29.78	(-1.01)	-45.16	(-1.05)
UNMP6	-7.57	(-0.85)	-6.29	(-0.54)	6.24	(1.42)	11.57	(2.86)
UNMP12	12.40	(1.94)	16.03	(1.76)	12.40	(1.94)	16.03	(1.76)
WAGE6	-6.49	(-2.67)	-8.48	(-4.67)	-7.52	(-2.84)	-9.57	(-4.16)
WAGE12	-7.20	(-0.78)	-7.62	(-0.87)	-7.20	(-0.78)	-7.62	(-0.87)
CPROF6			-0.03	(-0.01)			0.03	(0.02)
CPROF12			1.07	(0.22)			1.06	(0.22)
<i>R</i> ²	0.293		0.359		0.293		0.359	
Adj. <i>R</i> ²	0.127		0.135		0.127		0.135	
PKK Stat.	4.31*		6.75*		3.92*		5.45*	
Break date	1979:Q4		1981:Q2		1981:Q2		1980:Q4	
<i>DISP</i>								
Intercept	44.32	(5.50)	44.09	(4.93)	26.56	(3.32)	30.01	(3.09)
RTL6	-5.53	(-1.75)	-6.18	(-1.70)	-5.47	(-1.54)	-5.83	(-1.50)
RTL12	4.93	(0.03)	30.24	(0.17)	4.93	(0.03)	30.24	(0.17)
AUTO6	259.32	(2.03)	315.73	(2.34)	80.81	(1.22)	107.87	(1.28)
AUTO12	-173.34	(-1.44)	-201.85	(-1.62)	-173.34	(-1.44)	-201.85	(-1.62)
IP6	-457.10	(-1.66)	-512.69	(-1.58)	-857.15	(-3.05)	-850.98	(-2.67)
IP12	-398.14	(-6.36)	-336.67	(-5.14)	-398.14	(-6.36)	-336.67	(-5.14)
GDPX6	-438.14	(-0.98)	-468.59	(-0.93)	-959.74	(-3.13)s	-815.71	(-1.92)
GDPX12	-500.17	(-1.50)	-332.86	(-0.73)	-500.17	(-1.50)	-332.86	(-0.73)
UNMP6	25.00	(0.15)	-99.43	(-0.51)	-64.75	(-0.65)	-147.68	(-1.28)
UNMP12	-80.53	(-0.76)	-43.30	(-0.33)	-80.53	(-0.76)	-43.30	(-0.33)
WAGE6	92.46	(1.92)	98.11	(1.77)	34.10	(0.62)	45.57	(0.82)
WAGE12	-408.42	(-1.89)	-367.67	(-1.69)	-408.42	(-1.89)	-367.67	(-1.69)

Table 3 (continued)

CPROF6		127.04	(1.08)		127.04	(0.36)
CPROF12		-133.95	(-1.32)		-133.95	(-1.32)
R^2	0.535	0.558		0.535	0.558	
Adj. R^2	0.426	0.403		0.426	0.403	
PKK Stat.	4.11*	5.14*		3.84*	6.36*	
Break date	1980:Q2	1981:Q2		1982:Q4	1981:Q4	

Variable	Model 1		Model 2		Model 3	
	Coefficient	<i>t</i> -stat.	Coefficient	<i>t</i> -stat.	Coefficient	<i>t</i> -stat.

*Panel D: Induced uncertainty and dispersion of forecasts (the ASA-NBER Survey)**LU*

Intercept	1.16	(7.19)	1.00	(5.79)	1.05	(6.02)
$I_{\pi 1}$	-0.31	(-0.24)	0.01	(0.01)		
$I_{\pi 2}$					1.30	(1.82)
$I_{\pi 3}$	2.45	(1.94)				
$I_{\pi 4}$			3.99	(2.70)	3.46	(2.53)
$\sigma_{\pi 1}$	1.25	(0.65)	1.21	(0.63)		
$\sigma_{\pi 2}$					-1.95	(-0.93)
$\sigma_{\pi 3}$	-2.87	(-1.22)				
$\sigma_{\pi 4}$			-3.64	(-1.19)	-1.84	(-1.10)
R^2	0.033		0.059		0.058	
Adj. R^2	0.001		0.026		0.025	
PKK Stat.	1.25		1.23		1.09	
Break date	1982:Q1		1979:Q3		1979:Q3	

DISP

Intercept	17.44	(1.93)	13.85	(1.53)	14.46	(1.62)
$I_{\pi 1}$	-48.99	(-1.50)	-38.12	(-1.07)		
$I_{\pi 2}$					-34.14	(-0.76)
$I_{\pi 3}$	88.28	(1.66)				
$I_{\pi 4}$			108.75	(1.64)	125.61	(1.76)
$\sigma_{\pi 1}$	49.37	(0.99)	19.89	(0.47)		
$\sigma_{\pi 2}$					-36.64	(-0.59)
$\sigma_{\pi 3}$	-270.46	(-2.71)				
$\sigma_{\pi 4}$			-242.34	(-2.22)	-208.77	(-1.95)
R^2	0.150		0.135		0.153	
Adj. R^2	0.120		0.105		0.124	
PKK Stat.	4.39*		4.47*		4.67*	
Break date	1982:Q2		1982:Q3		1982:Q2	

Panel A reports the descriptive statistics of the standard deviations of the macro-economic forecasts of the Livingston Survey, calculated across forecasters. The survey data are from the Federal Reserve Bank of Philadelphia. We report the following variables: RTL, retail trade, AUTO, auto sales, both domestic and foreign, IP, industrial production index, GDPX, gross domestic product (prior to 1992 gross national product), UNP, civilian unemployment rate, WAGE, average weekly earnings in manufacturing, CPROF, corporate profit after taxes. We consider the 6- and 12-month ahead forecasts. To make them homogenous we use the growth rate, defined as a percentage change with respect to the previous month. For each variable we calculate the standard deviation across forecasts (i.e., responses in the survey per period). For each variable we report the year the variable was introduced in the survey, the descriptive statistics of our measure of dispersion of forecasts, the median number of responses per survey and the correlation between 6- and 12-month ahead forecasts. Panel B reports the descriptive statistics for the mean and the standard deviation (σ_{π}) of the probability of the recession as well as the descriptive statistics of the index of

(continued on next page)

Table 3 (continued)

uncertainty ($I_{\pi} = (1 - \pi)\pi$) as estimated by the participants of the ASA-NBER Survey of Professional Forecasters. The mean probability is calculated as the average of the probabilities of recession reported by the polled forecasters. The standard deviation (σ_{π}) is the standard deviation of the probabilities of recession across the polled forecasters. Both mean and standard deviation are expressed in percentage terms. To build the index of uncertainty, we first construct a measure of uncertainty for each forecaster (i.e., $(1 - \pi)\pi$) and then we average it out across forecasters. Panel C reports the results of the regression of induced uncertainty on the dispersions of forecasts in the Livingston Survey. In models 1 and 2 we use the original variables of the Livingston Survey. In models 3 and 4 the 12-month ahead forecasts are first orthogonalized by taking the residuals of the regressions of 12-month forecasts on the 6-month forecasts. In models 1 and 3 we use data from December 1966 (when forecasts of auto sales were added to the survey) to December 1998 (total 65 observations). In models 2 and 4 we use the data from June 1971 (date of addition of corporate profitability forecast) to December 1998. Estimations are done by using the Newey–West methodology with correction for autocorrelation up to 4 lags. *t*-Values are reported in parentheses. We also report the result of a Ploberger et al. (1989) (PKK) test of structural stability. A “*” signals that the PKK statistics rejects the null at the 5% level. Panel D reports the results of the regression of our proxies for induced uncertainty on the measures of dispersions of forecasts in the ASA-NBER Survey. Alternative specifications are estimated with different sets of variables.

283 4.3. An identification of learning uncertainty and dispersion of beliefs

284 We now have available two measures of uncertainty: one price-based and the
 285 other trade-based. Our claim is that the first proxies for learning uncertainty, while
 286 the second proxies for dispersion of beliefs. In order to prove this, we make use of
 287 measures of uncertainty that can be derived either from macro-economic surveys
 288 of economists and professional forecasters, or from micro-based variables.⁸

289 4.4. A macro-economic surveys

290 We consider the Livingston Survey and the ASA-NBER Survey. The Livingston
 291 Survey was started in 1946 by the late economist Joseph Livingston and is the oldest
 292 continuous survey of economists' expectations. It summarizes the forecasts of econ-
 293 omists from industry, government, banking, and academia. The Federal Reserve
 294 Bank of Philadelphia took over the responsibility for the survey in 1990. Every June
 295 and December, survey participants are asked to forecast a set of key macro-econ-
 296 omic variables for the end of the current month, two and four quarters ahead.⁹
 297 The number of participants varies over time, from 23 to 67 with the median around
 298 50.¹⁰ Descriptive statistics of the Livingston Survey are reported in Table 3, Panel
 299 A.

300 We select the variables that have the longest and most detailed coverage in terms
 301 of the numbers of forecasts: retail sales (RTL), auto sales (AUTO), industrial pro-
 302 duction (IP), GDP (GDPX), civilian unemployment rate (UNMP), average weekly

⁸ For a review of available sources see Croushore (1993) and references therein.

⁹ The full list can be found at <http://www.phil.frb.org/econ/liv/>.

¹⁰ The data of the Survey have been used extensively in the financial literature. For example, Lakonishok (1980) and Yoon and Edelstein (1989) studied biases in the Livingston Survey, and Gultekin (1983) focused on the influence of inflation forecast on returns.

303 earnings in manufacturing (WAGE), corporate profit after taxes (CPROF). For each
304 of these variables, we construct the standard deviations of the forecast values.¹¹

305 The ASA-NBER Survey of Professional Forecasters was initiated in 1968 and
306 generates the estimated probability of recession (π) from 1 to 4 quarters ahead. It
307 has a quarterly frequency. Descriptive statistics of it are displayed in Table 3, Panel
308 B, where we report the mean and standard deviation of the probability of recession
309 as estimated by participants of the ASA-NBER Survey. We derive two measures of
310 uncertainty from it. The first is constructed as the standard deviation of the forecasts
311 of the respondents to the Survey (σ_π). Given that this measure proxies for dispersion
312 of uncertainty across investors, we expect it to be more closely related to the disper-
313 sion of beliefs. A second measure is the average of the uncertainty of the respondents
314 (I_π). The uncertainty is constructed as $\pi(1 - \pi)$, where π is the probability of recession.
315 This measure peaks at 0.25 when the respondent is more uncertain ($\pi = 0.5$)
316 and goes to zero when he is certain ($\pi = 0$ or 1). Given that this measure is more related
317 to the uncertainty as perceived by each respondent, we think that it provides a
318 closer proxy for learning uncertainty. The joint use of these two measures allows us
319 to identify the two components of induced uncertainty.

320 We regress our measures of induced uncertainty on these survey-based measures
321 of uncertainty. In particular, in the case of the Livingston Survey, the explanatory
322 variable is the dispersion of forecasts, while in the case of the ASA-NBER Survey,
323 the explanatory variables contain *both* the dispersion of forecasts (σ_π) and the mea-
324 sures of pure uncertainty (I_π). We expect both our measures of induced uncertainty
325 to be related to the dispersion of forecasts derived from the Livingston Survey, as
326 this is related to both dispersion as well as uncertainty. However, in the case of
327 the ASA-NBER Survey, where we have separate proxies for dispersion and uncer-
328 tainty, the price-based measure should be mostly related to I_π , while the trade-based
329 measure should be mostly related to σ_π .

330 We report the results in Table 3, Panel C, for the Livingston Survey, and Panel D
331 for the ASA-NBER Survey. Given that the forecasts on corporate profits (CPROF)
332 are available for a shorter period, we consider a specification inclusive of them and
333 one where they are omitted. The results show that both measures of induced uncer-
334 tainty correlate with the dispersion of the forecasts contained in the Livingston Sur-
335 vey. The correlation is significant for both components of induced uncertainty. The
336 *Adjusted R*² is around 13% in the case of the price-based uncertainty and 43% in the
337 case of the trade-based uncertainty. In particular, the price-based measure is mostly
338 related to the dispersion of the six-month ahead forecasts of retail, auto sales and
339 average weekly earnings in manufacturing and to the 12-month ahead forecasts of
340 auto sales. The trade-based measure correlates mostly to the dispersion of six-month

¹¹ To make these forecasts homogenous, we first calculate the growth rate of the forecast values with respect to the last reported ones and then calculate their standard deviations. Also, in order to control for the strong collinearity between 6- and 12-month ahead forecasts, we also used the approach of Yoon and Edelstein (1989) and orthogonalize the 12-month ahead forecast on the 6-month ahead one. The residuals define the new variable.

341 ahead forecasts of auto sales and average weekly earnings in manufacturing and to
342 the 12-month ahead forecasts of industrial production.

343 The picture changes when we consider the measures of uncertainty based on the
344 ASA-NBER Survey. Here, we can run a horse race between dispersion and pure
345 uncertainty. Given that forecasts for different quarters are correlated, we consider
346 alternative specifications with different forecasts. The results agree with our identify-
347 ing restriction. The price-based measure of uncertainty is only related to the proxy
348 for pure uncertainty (I_π), while it is never related to the proxy for dispersion of fore-
349 casts (σ_π). The opposite is true for trade-based uncertainty that is related to the
350 proxy for dispersion of forecasts (σ_π), while it is never related to the proxy for pure
351 uncertainty (I_π). These results give a first indication that induced uncertainty is re-
352 lated to fundamental uncertainty and actually seems to lead it. Also, if we decom-
353 pose it into the two components, it appears that the price-based measure of
354 uncertainty is related mostly to pure uncertainty or risk, while the trade-based one
355 is related mostly to dispersion of beliefs.

356 4.5. A micro-based approach

357 Let us now investigate this issue in more detail by focusing exclusively on financial
358 data. To find proxies for the dispersion of beliefs and for risk that can be directly
359 derived from financial data is a difficult task – especially for the entire 1968–1998
360 period. Indeed, the only proxies the literature has identified are open interest and im-
361 plied volatility: the former as a proxy for dispersion of beliefs (Bessembinder et al.,
362 1996; Christensen and Prabhala, 1998) and the latter as a proxy for risk. However,
363 information on such variables is not available for the entire period under consider-
364 ation. We will therefore focus only on the sub-sample in which these variables are
365 available, and estimate:

$$E_t = \alpha + \beta OI_t + \gamma IV_t + \delta C_t + \varepsilon_t, \quad (2)$$

368 where E_t is our measure of induced uncertainty (i.e., LU_{*t*} or DISP_{*t*}) and OI_{*t*} and IV_{*t*}
369 are, respectively, open interest and implied volatility on all financial and stock index
370 futures and options traded on CBOT. C_t represents a set of control variables.¹²

371 The results, reported in Table 4, show that price-based uncertainty is related *only*
372 to implied volatility and trade-based uncertainty *only* to open interest. These results
373 are very robust and hold in all the tested specifications. The Adjusted R^2 is high (12%
374 for the case of the price-based measure of uncertainty and 53% for the case of the
375 trade-based measure of uncertainty) even in the case where no control variables have
376 been included. The implication is that trade-based uncertainty is related mostly to
377 dispersion of beliefs, while price-based uncertainty is related mostly to risk. This is
378 consistent with the results in the previous section.

379 As additional robustness check, we also consider a measure of dispersion based on
380 analysts' forecasts. This variable (IBESDISP) is the average dispersion of next fiscal
381 quarter EPS forecast (scaled by mean EPS) as reported by IBES on the last day of
382 the month.¹³ We estimate how our measures (LU and DISP) are related to it. LU is
383 not related to IBESDISP, while DISP is strongly related to it. This supports our

Table 4
Induced uncertainty and financial uncertainty

Variable	LU		DISP		DISP		DISP	
	Coefficient	<i>t</i> -stat.	Coefficient	<i>t</i> -stat.	Coefficient	<i>t</i> -stat.	Coefficient	<i>t</i> -stat.
Intercept	0.73	(3.25)	0.72	(3.06)	-7.06	(-1.19)	-7.07	(-1.36)
IV	0.03	(6.23)	0.03	(5.95)	-0.13	(-0.61)	-0.07	(-0.49)
OI	0.01	(0.13)	0.03	(1.46)	15.79	(5.83)	12.04	(5.77)
IBESDISP			-0.03	(-0.44)			129.5	(3.74)
R^2	0.122		0.124		0.538		0.552	
Adj. R^2	0.111		0.108		0.532		0.544	

This table reports the results of the regression of our proxies for induced uncertainty on implied volatility (IV) and open interest (OI). IV is the implied volatility index and OI is the open interest on all the financial and stock index futures and options traded on the CBOT. IBESDISP is average standard deviation of next fiscal quarter EPS forecast (scaled by mean EPS) as reported by IBES as of last day of the month. Only observations for firms with more than two analysts were considered. Other variables are defined as in Table 4. The estimation period is 1986:01–1998:12. All the estimates for DISP are multiplied by 100. The coefficient on OI has been multiplied by 10^6 .

384 working assumption that our trade-based measure is proxying for dispersion of
385 beliefs.

386 5. Is induced uncertainty priced?

387 We are now ready to study whether both measures of uncertainties are priced. Gi-
388 ven that we have ascertained in the previous section that these two measures of in-
389 duced uncertainty proxy for different dimension of uncertainty, from now on, we will
390 call learning uncertainty the price-based measure of uncertainty and dispersion of
391 beliefs the trade-based one. We consider a conditional specification. Let us see it
392 in more detail. We assume that there are M portfolios of stocks, with $j = 1, \dots, M$
393 and N factors that affect returns, with $n = 1, \dots, N$. The factors (F_n) include the stan-
394 dard ones (i.e., the three Fama and French factors and the Carhart momentum fac-
395 tor) and our measures of induced uncertainty (i.e., LU and DISP). The standard
396 factors are derived from K. French's web page. The test is based on the conditional
397 framework laid out by Dumas and Solnik (1995). We outline here the main restric-
398 tions of this approach, referring to their paper for further details:

$$E[R_{j,t}|\Omega_{t-1}] = \sum_{n=1}^N \lambda_{n,t-1} \text{cov}[R_{j,t}, F_{n,t}|\Omega_{t-1}], \quad (3)$$

402 where $R_{j,t}$ is the return on the j th risky portfolio in excess of the riskless rate from
403 time $t - 1$ to t , $F_{n,t}$ represents the n th factor (or source of risk) for the same period,
404 $\lambda_{n,t-1}$ is the price of the n th source of risk and Ω_{t-1} is the information set investors
405 use in order to make their portfolio decision. Let us see how Eq. (3) generates a set of
406 testable restrictions. We define m_t as the marginal rate of substitution between re-
407 turns at date t and at date $t - 1$ and $R_{f,t-1}$, as the conditionally risk-free rate for

408 the period starting at $t - 1$. Standard theory (Cochrane, 2001) tells us that the first
409 order conditions of the portfolio choice problem are:

$$\begin{cases} E[m_t(1 + R_{f,t-1})|\Omega_{t-1}] = 1, \\ E[m_t R_{j,t}|\Omega_{t-1}] = 0. \end{cases} \quad (4)$$

413 By using Eqs. (3) and (4), Dumas and Solnik (1995) show that m_t can be defined as

$$m_t = \left[1 - \lambda_{o,t-1} - \sum_{n=1}^N \lambda_{n,t-1} F_{n,t} \right] / (1 + R_{f,t-1}), \quad (5)$$

417 where $\lambda_{o,t-1}$ appears as a “way of ensuring that the system of Eq. (4) is satisfied.”
418 The system of Eq. (4) defines the orthogonality conditions of the econometric
419 specification.

420 We assume that the information set Ω_{t-1} is made of a vector of instrumental vari-
421 ables \mathbf{Z}_{t-1} . These variables were used by Ferson and Harvey (1991, 1999), Dumas
422 and Solnik (1995). They include: a constant, the junk premium spread between Moo-
423 dy’s Baa and Aaa corporate bond yields (JUNK), the dividend yield of the S&P 500
424 index (DIVY), the term premium spread between a 10-years and a one-year Treasury
425 bond yield (TERM), the spread between the three-month and one-month T-bill
426 (HB3), January dummies, the return of the market portfolio (R_{MKT}) and the lagged
427 one-month T-bill yield (TB30D). We also assume a linear relationship between such
428 state variables and the λ s, such that: $\lambda_{o,t-1} = -\mathbf{Z}'_{t-1}\boldsymbol{\delta}$ and $\lambda_{n,t-1} = \mathbf{Z}'_{t-1}\boldsymbol{\phi}_n$, where $\boldsymbol{\delta}$ and
429 $\boldsymbol{\phi}$ s are time-invariant vectors of weights. In particular,
430 $\boldsymbol{\delta} = [\delta_c, \delta_J, \delta_D, \delta_T, \delta_H, \delta_{Ja}, \delta_M, \delta_Y]'$, where $\delta_c, \delta_J, \delta_D, \delta_T, \delta_H, \delta_{Ja}, \delta_M, \delta_Y$ are the sensi-
431 tivities of the intercept term ($\lambda_{o,t-1}$) to the information variables in the order they
432 have been previously defined. Analogously, $\boldsymbol{\phi} = [\phi_c, \phi_J, \phi_D, \phi_T, \phi_H, \phi_{Ja}, \phi_M, \phi_Y]'$,
433 where $\phi_c, \phi_J, \phi_D, \phi_T, \phi_H, \phi_{Ja}, \phi_M, \phi_Y$ are the sensitivity of the price of the n th source
434 of risk ($\lambda_{n,t-1}$) to the information variables in the order they have been previously
435 defined. We define the innovation ξ_t in the marginal rate of substitution as

$$\xi_t = 1 - m_t(1 + R_{f,t-1}). \quad (6)$$

438 Using Eq. (5), we can write:

$$\xi_t = -\mathbf{Z}'_{t-1}\boldsymbol{\delta} + \sum_{n=1}^N \mathbf{Z}'_{t-1}\boldsymbol{\phi}_n F_{n,t}. \quad (7)$$

442 Let us define $h_{j,t} = R_{j,t} - R_{j,t}\xi_t$. The pricing conditions of Eq. (4) imply the following
443 orthogonality conditions:

$$\begin{cases} E[\xi_t|\Omega_{t-1}] = 0 \\ E[\mathbf{h}_t|\Omega_{t-1}] = 0. \end{cases} \quad (8)$$

447 where \mathbf{h}_t is the vector that contains the $h_{j,t}$ stacked for all the considered portfolios.
448 Defining the vector of residuals $\boldsymbol{\varepsilon}_t = (\xi_t, \mathbf{h}_t)$, and combining the orthogonality condi-
449 tions, we have

$$E[\boldsymbol{\varepsilon}_t|\mathbf{Z}_{t-1}] = 0, \quad (9)$$

453 which implies the restriction $E[\mathbf{e}'_i \mathbf{Z}'_{i-1}] = 0$, whose sample version is

$$454 \quad \mathbf{Z}' \boldsymbol{\varepsilon} = 0. \quad (10)$$

457 Here \mathbf{Z} is $T \times Q$ matrix, where Q is the number of instruments that are employed and $\boldsymbol{\varepsilon}$
 458 is $T \times (1 + M)$ matrix, where T is the number of observations over time and M is the
 459 number of portfolios we use (25 in the book-to-market and size specification and 30
 460 in the industry specification). This represents a total of $Q \times (1 + M)$ conditions. The
 461 moments conditions of Eq. (10) represent our testable restrictions. Notice that λ s
 462 have now been replaced by combinations of instruments (\mathbf{Z}), such that
 463 $\lambda_{o,t-1} = -\mathbf{Z}'_{t-1} \boldsymbol{\delta}$ and $\lambda_{n,t-1} = \mathbf{Z}'_{t-1} \boldsymbol{\phi}_n$, where $\boldsymbol{\delta}$ and $\boldsymbol{\phi}$ s. This allows us to estimate $\boldsymbol{\delta}$
 464 and $\boldsymbol{\phi}$ directly, without estimating the λ s. The intuition behind the restrictions that
 465 link λ s to \mathbf{Z} is that risk premia (i.e., λ s) can be predicted using a set of observable
 466 variables. Rewritten this way, the estimation of Eq. (10) is just the estimation of a
 467 series of orthogonal restrictions with robust standard errors, performed using stan-
 468 dard GMM procedures. By minimizing the average deviation from these moment
 469 conditions, we derive the best estimates of the parameters $\boldsymbol{\delta}$ and $\boldsymbol{\phi}$. The deviations
 470 in the various moment conditions are weighted by a weighting matrix, W , which
 471 is the inverse of a consistent covariance matrix of sample moment conditions. Given
 472 that the main variables of interest (LU and DISP) have been constructed as a by-
 473 product of a previous estimation, the GMM estimates may be subject to a generated
 474 regression problem. To address this issue, we use the parametric bootstrap method
 475 developed by Smith and McAleer (1993). In Tables 5 and 6, next to the ordinary sta-
 476 tistics, we also report the value of the bootstrapped statistics.

477 We will test whether the factors are priced by using the test of over-identifying
 478 restrictions for Eq. (10). That is, under the null that the system (8) hold, the value
 479 of the quadratic form of Eq. (10) is asymptotically distributed as a χ^2 with degrees
 480 of freedom equal to the number of orthogonality conditions minus the number of
 481 parameters (Dumas and Solnik, 1995).

482 We perform our analysis on portfolios. Following the Fama and French ap-
 483 proach, we aggregate the stocks in either 25 value-weighted portfolios on the basis
 484 of the book-to-market and size classification, or 30 portfolios on the basis of four-
 485 digit SIC codes. We differ from the standard procedure in that we limit our universe
 486 of securities by using only securities that have volume reported. This effectively re-
 487 duces our sample during the 1963–1982 to NYSE/AMEX stocks. Extensive robust-
 488 ness checks (available upon request) show that the results are not sensitive to the
 489 subtraction of NASDAQ firms from our sample. The summary statistics of the 25
 490 size/book-to market portfolios and 30 industry portfolios are reported in Table 1.
 491 We can see that even if the sample is different (we used the period 1963–1998, while
 492 Fama and French (1993) used the period 1963–1990) the main statistics are similar to
 493 the Fama and French ones.

494 Following the standard methodology, we construct the additional factors as the
 495 one-period ahead change of our measures of informational uncertainty. The basic
 496 factors are the Fama and French factors (Market, HML and SMB) augmented by

Table 5
 Conditional tests of pricing

	δ	ϕ_{MKT}	ϕ_{HML}	ϕ_{SMB}	ϕ_{UMD}	ϕ_{LU}	ϕ_{DISP}
<i>Panel A: 25 book-to-market and size portfolios</i>							
Constant	0.42 (5.76)***	-0.03 (-0.02)	5.83 (1.99)	-7.23 (-2.87)*	-0.02 (-0.77)	15.65 (2.36)	0.00 (0.29)
JUNK	103.31 (1.31)	3175.76 (1.83)	-2519.57 (-0.91)	-5086.30 (-1.65)	-51.55 (-2.07)	7395.14 (1.11)	-52.51 (-3.83)***
DIVY	-72.92 (-1.76)	1082.00 (1.22)	-119.12 (-0.08)	5477.28 (3.73)**	30.65 (2.09)	-5019.34 (-1.83)	2.94 (0.53)
TERM	3.91 (3.54)**	52.78 (2.38)	21.99 (0.51)	-109.09 (-3.56)**	-1.55 (-4.86)***	-162.42 (-1.68)	0.08 (0.71)
HB3	21.04 (3.23)**	583.27 (4.64)***	-283.80 (-1.05)	-75.79 (-0.27)	8.31 (4.68)***	-571.80 (-0.89)	-0.42 (-0.39)
DUMMY _{January}	0.17 (2.72)	-0.14 (-0.14)	4.56 (2.03)	-4.15 (-2.74)**	0.01 (0.50)	-11.70 (-2.18)*	0.00 (0.45)
R_{MKT}	-0.81 (-1.30)	-11.90 (-1.18)	7.89 (0.60)	153.10 (10.15)***	0.34 (2.54)*	71.97 (2.55)**	0.12 (3.14)**
TB30D	23.21 (1.77)	-1133.33 (-3.68)**	-94.29 (-0.19)	-811.48 (-1.66)	-5.83 (-1.51)	953.09 (0.82)	3.38 (1.80)
$\chi^2 = 156.250^{***}$; degrees of freedom: 152; right-tail p-value 0.3899							
<i>Panel B: 30 industry portfolios</i>							
Constant	-0.02 (-0.18)	-0.08 (-0.02)	-8.06 (-1.21)	-22.49 (-3.86)**	-0.05 (-0.79)	-5.08 (-0.37)	0.09 (4.96)***
JUNK	-285.17 (-2.04)	9559.47 (2.43)	10899.06 (1.57)	6507.00 (1.13)	-288.24 (-5.26)***	87074.76 (5.41)***	-92.48 (-3.84)**
DIVY	-16.62 (-0.24)	3504.76 (1.73)	-1654.85 (-0.55)	614.14 (0.24)	126.85 (4.21)**	-19023.10 (-2.95)	-5.42 (-0.55)
TERM	-1.61 (-0.75)	78.49 (1.30)	259.11 (2.45)*	47.90 (0.57)	-0.17 (-0.21)	370.82 (1.96)	-0.48 (-2.25)

HB3	11.95 (1.24)	1455.43 (4.42)***	-2141.03 (-3.72)**	-2000.29 (-3.76)**	20.06 (6.01)***	-6105.15 (-3.92)***	1.09 (0.56)
DUMMY _{January}	-0.23 (-1.60)	-2.04 (-0.82)	0.40 (0.06)	22.36 (5.38)***	0.02 (0.83)	39.86 (2.77)**	-0.02 (-1.17)
R_{MKT}	-0.08 (-0.06)	12.09 (0.58)	39.60 (1.11)	222.55 (7.90)***	-0.12 (-0.34)	-11.98 (-0.24)	0.07 (0.67)
TB30D	-0.57 (-0.02)	-3017.96 (-4.01)**	1024.48 (0.87)	2187.99 (2.07)	-4.82 (-0.58)	165.02 (0.06)	1.57 (0.44)

$\chi^2 = 138.851^{***}$; degrees of freedom:192; right-tail p -value 0.9985

This table reports the Generalized Method of moments tests of the moment conditions of the conditional pricing test (Eq. (9) in the text). Panel A reports estimates for the 25 book-to-market and size portfolios; Panel B reports estimates for the 30 industry portfolios. The vectors, δ , ϕ_{MKT} , ϕ_{HML} , ϕ_{SMB} , ϕ_{UMD} , ϕ_{LU} , ϕ_{DISP} , contain the coefficients of the linear relationship between λ , λ_{MKT} , λ_{HML} , λ_{SMB} , λ_{UMD} , λ_{LU} , λ_{DISP} and the vector of instruments, \mathbf{Z} . (MKT, HML and SMB are the three Fama and French factors, while UMD is the Carhart Momentum factor). Excess rates of return are coded as 1 for a 1% return per month. The factors that proxy for induced uncertainty have been previously orthogonalized by regressing them on the FF factors. The instrumental variables that are expressed as yields or returns are coded as 0.01 for a 1% return per month. The set of instrumental variables include a constant, the one month T-bill yield (TB30D), the dividend yield of the S&P 500 index (DIVY), the term premium-spread between a 10-year and a one-year Treasury bond yield (TERM), the junk premium-spread between Moody's Baa and Aaa corporate bond yields (JUNK), the difference between the one-month returns of a three-month and one-month T-bill (HB3), the excess return of the market (R_{MKT}), and a January dummy that takes value 1 for January and 0 otherwise. All the estimations are done for the period 1964:07–1998:12 (number of observations = 414). We report the estimated coefficient in the first row of each cell. t -statistics is reported below, in parentheses. The last row of each panel reports the χ^2 , the number of degrees of freedom and the associated p -value. Asterisks denote the significance level based on Monte-Carlo Bootstrapping. * Indicates significance on 10% level, ** on 5%, and *** on 1% level. Asterisks on χ^2 denote the p -value based on the number of times the χ^2 in the Monte-Carlo replications exceeded the corresponding χ^2 value.

PROOF

Table 6
Hypothesis testing

Factors in unrestricted model	Null hypothesis	D stat.	Degrees of freedom	p-value
<i>Panel A: LU and DISP not included among the information variables</i>				
<i>25 book-to-market and size portfolios</i>				
R_{MKT} , LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	411.75	16	0.000***
	$\phi_{LU} = 0$	195.37	8	0.000***
	$\phi_{DISP} = 0$	262.92	8	0.000***
R_{MKT} , HML, SMB, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	104.84	16	0.000***
	$\phi_{LU} = 0$	35.56	8	0.000***
	$\phi_{DISP} = 0$	67.57	8	0.000***
R_{MKT} , HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	62.16	16	0.000***
	$\phi_{LU} = 0$	27.06	8	0.001***
	$\phi_{DISP} = 0$	41.34	8	0.000***
<i>30 industry portfolios</i>				
R_{MKT} , LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	318.09	16	0.000***
	$\phi_{LU} = 0$	135.09	8	0.000***
	$\phi_{DISP} = 0$	74.68	8	0.000***
R_{MKT} , HML, SMB, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	194.34	16	0.000***
	$\phi_{LU} = 0$	75.42	8	0.000***
	$\phi_{DISP} = 0$	95.48	8	0.000***
R_{MKT} , HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	121.45	16	0.000***
	$\phi_{LU} = 0$	41.74	8	0.000***
	$\phi_{DISP} = 0$	62.61	8	0.000***
<i>Panel B: LU and DISP included among the information variables</i>				
<i>25 book-to-market and size portfolios</i>				
R_{MKT} , LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	633.04	20	0.000***
	$\phi_{LU} = 0$	382.02	10	0.000***
	$\phi_{DISP} = 0$	299.88	10	0.000***
R_{MKT} , HML, SMB, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	714.77	20	0.000***
	$\phi_{LU} = 0$	314.95	10	0.000***
	$\phi_{DISP} = 0$	136.26	10	0.000***
R_{MKT} , HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	199.01	20	0.000***
	$\phi_{LU} = 0$	107.29	10	0.000***
	$\phi_{DISP} = 0$	92.57	10	0.000***
<i>30 industry portfolios</i>				
R_{MKT} , LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	343.09	20	0.000***
	$\phi_{LU} = 0$	44.26	10	0.000***
	$\phi_{DISP} = 0$	298.41	10	0.000***
R_{MKT} , HML, SMB, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	273.45	20	0.000***
	$\phi_{LU} = 0$	125.29	20	0.000***
	$\phi_{DISP} = 0$	153.02	20	0.000***
R_{MKT} , HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	191.44	20	0.000***
	$\phi_{LU} = 0$	83.85	10	0.000***
	$\phi_{DISP} = 0$	97.53	10	0.000***

Table 6 (continued)

Factors in unrestricted model	Null hypothesis	Degrees of freedom	Periods			
			1964–1981		1982–1998	
			<i>D</i> stat.	<i>p</i> -value	<i>D</i> stat.	<i>p</i> -value
<i>Panel C: Hypothesis testing in sub-periods</i>						
<i>25 book-to-market and size portfolios</i>						
R_{MKT} , LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	20	1966.16	0.000***	1041.36	0.000***
	$\phi_{LU} = 0$	10	342.16	0.000***	417.12	0.000***
	$\phi_{DISP} = 0$	10	916.16	0.000***	583.16	0.000***
R_{MKT} , HML, SMB, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	20	450.59	0.000***	628.76	0.000***
	$\phi_{LU} = 0$	10	82.59	0.000***	143.76	0.000***
	$\phi_{DISP} = 0$	10	271.59	0.000***	372.76	0.000***
R_{MKT} , HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	20	316.80	0.000***	394.54	0.000***
	$\phi_{LU} = 0$	10	174.42	0.000***	56.66	0.000***
	$\phi_{DISP} = 0$	10	154.04	0.000***	277.94	0.000***
<i>Industry portfolios</i>						
R_{MKT} , LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	20	311.94	0.000***	706.41	0.000***
	$\phi_{LU} = 0$	10	48.59	0.000***	422.14	0.000***
	$\phi_{DISP} = 0$	10	213.53	0.000***	168.67	0.000***
R_{MKT} , HML, SMB, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	20	684.34	0.000***	218.7	0.000***
	$\phi_{LU} = 0$	10	195.04	0.000***	43.34	0.000***
	$\phi_{DISP} = 0$	10	195.14	0.000***	121.41	0.000***
R_{MKT} , HML, SMB, UMD, LU, DISP	$\phi_{LU} = 0, \phi_{DISP} = 0$	20	225.69	0.000***	201.02	0.000***
	$\phi_{LU} = 0$	10	49.12	0.000***	87.45	0.000***
	$\phi_{DISP} = 0$	10	63.16	0.000***	81.41	0.000***

We report the results of the tests of whether induced uncertainty (LU, DISP) is priced. The factors that proxy for induced uncertainty have been previously orthogonalized by regressing them on the FF factors. The first and second column describe the hypothesis that is tested and the alternative. The third, fourth and fifth columns report, respectively, the Newey and West (1987) *D*-statistics, its number of degrees of freedom and the associated *p*-value. This statistics is distributed like as a χ^2 . Asterisks denote the significance level based on Monte-Carlo bootstrapping. * Indicates significance on 10% level, ** on 5%, and *** on 1% level. Panel A reports the results for the specification in which the vector of information variables contains a constant, the junk bond premium (JUNK), the difference between the one-month returns of a three-month T-bill and the return of a one-month T-bill (HB3), the term premium (TERM), the dividend yield of the S & P500 index (DIVY), the excess market return (R_{MKT}), the one-month T-bill rate (TB30D), and a January Dummy. Panel B reports the results when the lagged values of LU and DISP are included among the information variables. Panel C reports the results of testing whether LU and DISP are priced in the sub-periods. They are: 1964:07–1981:12 and 1982:01–1998:12.

497 Carhart momentum factor.¹⁴ We also consider an alternative specification where the
 498 restricted model contains just the market portfolio and the unrestricted one contains
 499 the market portfolio and the two factors that proxy for induced uncertainty. We also
 500 consider specifications where either of the two factors proxying for induced uncer-
 501 tainty is separately considered. We report the values of the estimated coefficients
 502 (ϕ_{fs}) in Table 5, Panels A and B, for the size/book-to-market and industry portfo-
 503 lios, respectively. To assess the incremental explanatory power of the factors proxy-
 504 ing for induced uncertainty, we compare the unrestricted specification (inclusive of
 505 the factors proxying for induced uncertainty) to the restricted one (not containing
 506 these factors). The nice feature of this approach is the fact that the restricted and
 507 the unrestricted specifications are nested. We can therefore use the restricted specifi-
 508 cation as the null hypothesis and the unrestricted one as the alternative and test
 509 whether the restrictions of setting the ϕ_{fs} to zero in Eq. (7) for the additional two
 510 factors are rejected. The test is a Newey–West Difference test, based on the difference
 511 between the χ^2 s of the restricted and unrestricted model (Newey and West, 1987;
 512 Eichenbaum et al., 1988). Formally, this corresponds to testing:

$$H_0 : \phi_{LU} = \phi_{DISP} = 0 \quad \text{and} \quad H_A : \phi_{LU} \neq 0 \neq \phi_{DISP}. \quad (11)$$

515 We proceed as follows. First, we calculate the χ^2 in the unrestricted model – i.e.,
 516 six-factor specification. Then, we drop the factors proxying for induced uncertainty,
 517 we reestimate the model using a 4-factor specification (i.e., we impose $\phi_{LU} = \phi_{DISP} = 0$)
 518 using the same weighting matrix W as the one estimated in the unrestricted
 519 case (six factors) and we calculate the χ^2 in the restricted model. The difference be-
 520 tween the two χ^2 s is a new χ^2 with the number of degrees of freedom equivalent
 521 to the number of new restrictions. This allows us to compare the restricted model
 522 with the unrestricted one. If the χ^2 in the restricted model is significantly higher than
 523 the χ^2 in the unrestricted one, the null of no pricing of the induced factors is rejected.

524 The tests of the hypothesis of pricing are reported in Table 6, Panel A, for both
 525 the size and book-to-market and industry portfolios. In all the cases, both LU and
 526 DISP factors are priced, both jointly and separately. The χ^2 in the restricted model is
 527 always significantly higher than the χ^2 in the unrestricted one and the null is always
 528 rejected at any confidence level. Also in this case, we address the issue of generated
 529 regressors by using the parametric bootstrap approach (Smith and McAleer, 1993).

530 Up to now, we have implicitly assumed that induced uncertainty does not enter
 531 investors' information sets. However, it is possible that investors directly condition
 532 on the level of information uncertainty in order to make their portfolio decisions.
 533 That is, investors use past measures of information uncertainty in order to derive
 534 forecasts of future uncertainty and condition their portfolio decisions on it. In this
 535 case, part of the explanatory power of the additional factors could be due simply
 536 to the fact that they capture the time variation of investors' information sets. As Fer-
 537 son and Harvey (1999) pointed out, "simple proxies for time variation in expected
 538 returns, based on common instruments are also significant cross-sectional predica-

¹⁴ The factors that proxy for induced uncertainty have been previously orthogonalized by regression on the FF factors.

539 tors of returns. The ability of these variables to explain the cross-section of returns
540 provides a powerful rejection of the Fama and French model as a conditional asset
541 pricing model.” The conditional framework allows us to assess whether induced
542 uncertainty is actually priced or if it only enters as an additional conditioning vari-
543 able that affects investors’ information sets.

544 We therefore estimate again the conditional specification, including our measures
545 of induced uncertainty among the information variables as well as among the fac-
546 tors. Induced uncertainty is now part of investors’ information sets (Ω_{t-1}) as well
547 as part of the factors that affect the price of risk (λ s). If the components of induced
548 uncertainty turn out to be significant *even after conditioning on their past values*, this
549 provides direct evidence that these components of induced uncertainty are an addi-
550 tional source of risk against which the market hedges.

551 We report the tests of the hypothesis of pricing for this specification in Table 6,
552 Panel B, for both the size and book-to-market and industry portfolios. Also in this
553 case, for all the specifications – i.e. one- and four-factor models – the χ^2 in the re-
554 stricted model is always significantly higher than the χ^2 in the unrestricted one. In
555 all the cases, the factors that proxy for induced uncertainty are priced, both jointly
556 and separately.

557 These results provide strong support for the theoretical models (Brennan, 1998;
558 Brennan and Xia, 2001b) that argue that investors hedge against information uncer-
559 tainty. Indeed, not only are informational uncertainty and dispersion of beliefs
560 priced, but they also affect the price of risk. In other words, investors hedge against
561 their fluctuations.

562 Finally, we test whether the pricing impact of induced uncertainty on prices differs
563 in different periods. We focus on two periods: 1964–1980 and 1981–1998.

564 The results are reported in Table 6, Panel C. As before, we also apply the para-
565 metric bootstrap method to control for the generated regressor problem and report
566 the value of the bootstrapped statistics. The findings are consistent with the earlier
567 results and show that both learning uncertainty and dispersion of beliefs are priced.

568 As additional robustness check we also considered a specification in which the dis-
569 persion of beliefs is proxied by the dispersion of analysts forecasts, constructed by
570 aggregating IBES recommendations. Given the more limited availability of this var-
571 iable, the sample period is shorter. The results (not reported) are consistent with the
572 previous ones. Dispersion of beliefs is conditionally priced in the case of the industry
573 portfolios as well as in the case of the book-to-market ones. In particular, in the case
574 of the book-to-market portfolios, the sensitivity of the price of the dispersion of be-
575 liefs risk to the information variables is significant in the case of the junk premium
576 spread, the dividend yield, the term premium spread, the January dummy and the
577 return of the market portfolio. In the case of the industry portfolios, the sensitivity
578 of the price of the dispersion of beliefs risk to the information variables is significant
579 in the case of the junk premium spread, the dividend yield, the spread between the
580 three-month and one-month T-bill, and the January dummy. Moreover, for all the
581 specifications – i.e., one- and four-factor models – the χ^2 in the restricted model is
582 always significantly higher than the χ^2 in the unrestricted one. That is, the dispersion

583 of beliefs risk is priced. These results suggest that, while the standard factors capture
584 expectations about future fundamentals and innovations about them, our measures
585 of induced uncertainty *capture the dispersion around them*.

586 6. Conclusion

587 We empirically investigate “induced” uncertainty and constructed proxies for its
588 two main components: one related to learning uncertainty and one related to disper-
589 sion of beliefs. We show that induced uncertainty represents a component of the risk
590 premia that is separate from the fundamental component. We find compelling evi-
591 dence that both learning uncertainty and dispersion of beliefs are conditionally
592 priced.

593 These results have implications from a theoretical standpoint as well as from a
594 more empirical one. The identification of the return component related to induced
595 uncertainty may help not only to explain asset returns, but also to deal with unre-
596 solved puzzles or apparent market anomalies such as the day-of-the-week effect or
597 momentum. Moreover, it sheds some light on the determinants of trading volume
598 and on its potential explanatory power to predict stock returns. A further promising
599 extension would be to investigate the out-of-sample power of systematic trade in or-
600 der to predict future asset returns.

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