Competition in Markets for Information

Andrei Simonov*
INSEAD

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Abstract

Competition among sellers of information in a noisy rational expectations equilibrium is considered. Traders' preferences for information are explicitly characterized. It is shown that the competition on market for information makes providers of financial information price their products in a way that leads traders to purchase *all* signals available. If signals are substitutes, competition pushes the price of information lower than that in monopolistic settings. However, if signals are complements, the price of an individual signal in duopoly actually exceeds the one in monopolistic settings, and information producers are involved in tacit collusion. Externalities of information lead to counterintuitive results showing that (a) efficiency of the competitive market for information (as measured by quality of signals offered for sale) is no better than in monopolistic setting, and (b) competition leads to no improvement on the part of traders as providers of financial information are still able to appropriate all of the consumer surplus.
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1. Introduction

Selling information to aid financial decisions is an increasingly important part of modern business. The industry that provides information is rapidly growing as is the usage of the information by small investors. Both the regulators and the general public seem to believe that such growth is a positive development. Presumably, such services should increase the amount of information available to investors and make the market more efficient. The aim of this paper is to shed some light on the role of competition and the choice of information production technology in the information selling process. I will show that the hopes of the investors are exaggerated. The externalities that the information as a commodity generates result in a number of counter-intuitive conclusions. In particular, I show that the competition among the providers of financial information may result in a deterioration of information quality and lower the informational efficiency of the market. The competing providers are able to extract the same amount of rent as a monopolist does, leading to no welfare improvement on the part of the traders.

We know very little about the mechanics of the competitive market for information. Beginning with the papers of Grossman and Stiglitz (1980) and Admati and Pfleiderer (1986, 1987, 1990), the modeling of information selling has been set mainly in a monopolistic context. Admati and Pfleiderer consider a simple homogeneous market for information in a noisy rational expectations equilibrium framework. They show that if the noise is not very large, it might not make sense for the seller to sell his private information directly (for example, through a newsletter), but to do it indirectly (via a mutual fund).[1] If the indirect sale of information is not possible, it does make sense for a monopolist to either limit the number of units of information he sells, or to add some noise to his information, or both; see Admati and Pfleiderer (1986). The major trade-off is between selling more copies of the newsletter and controlling dissemination of information through prices. Note that in models with a single information provider *à la* Grossman and Stiglitz (1980), the information provider appropriates all of the informed traders' surplus.

However, in real life, a monopolistic market for information is rarely observed.

[1]Kane and Marks (1990) compare the direct and indirect sale of information in the presence of borrowing or short sale constraints. Their results seem to indicate that, under constraint, the direct sale of information (e.g., a newsletter delivery system) will be at least as efficient as an indirect one. Similarly, Brennan and Chordia (1993) consider optimal fees for brokerage information services. They show that a payment mechanism based on extreme realizations of the signal might dominate both direct and indirect sales of information. However, their results rely on the assumption that the propagation of information through prices is the same for both direct and indirect sales of information.

Casual observation tells us that, for the majority of $S\&P$ 500 companies, there are multiple sources of information available. For example, there exist more than a dozen newsletters directed toward investors to the *Fidelity* family of mutual funds. Thus, it seems that to consider competition in information markets is not the only natural way to extend the Admati and Pfleiderer (1986) framework, but that it is necessary to understand real life phenomena.

The difficulty of modeling information selling arises from the externalities of information as a commodity. First, the information is more valuable when fewer traders know about it. Second, information propagates through the price system. High quality information induces risk averse traders to trade more aggressively. If information about the asset payoff is used more aggressively, it will be revealed through prices more strongly, making the initial signal less valuable. Finally, as was pointed out by Allen (1986), combining different types of information may lead to improving overall precision by providing additional information to the trader. As a result, the information market, with a variety of signals, should be more efficient than one with monopolistic information providers.

Fishman and Hagerty (1995), Sabino (1993), Shin (1998), Biais and Germain (1997) and Germain (1998) consider the question of commitment to certain types of trade by the information provider.[2] Fishman and Hagerty (1995) showed that selling information might be a strategic act of an informed trader. Instead of simply trading on his information, an informed trader sells the signal to a chosen number of traders. As a result, all informed traders rationally trade less aggressively. However, information sellers appropriate the surplus of new buyers and, in addition, make trading profit. Fishman and Hagerty (1995) show that combined profit does indeed dominate the equilibrium without selling information. The model itself is close to those of Kyle (1985) and Admati and Pfleiderer (1988).

Biais and Germain (1997) and Germain (1998) consider the possibility of adding noise as a way of softening the effect of competition among information providers in the Glosten and Milgrom (1985) framework. In the Germain (1998) model, adding noise provides camouflage for informed traders who can then trade on the market. Both Fishman and Hagerty (1995) and Germain (1998) consider information selling as a way to maximize the profitability of informed trader trades. In a way, it is close to the manipulation idea of Benabou and Laroque (1992). As there, the agent choose to provide signals that are designed to maximize subsequent trading profits. Note that none of these models takes into account the effect of the "natural" constraint of the number of informed traders.

Thus, the bulk of the literature considers information selling in monopoly

[2]Models of Sabino (1993) and Shin (1998) are almost indistinguishable, and are very close to those of Fishman and Hagerty (1995). In the following discussion I will refer only to the latter.

settings. The few papers that address the role of competition in informational markets do so under the simplified assumption of either infinite capacity of the market or by assuming that information does not propagate through prices. The present paper tries to analyze the competition in information markets within a self-consistent framework that takes into account both effects. The main questions addressed in this paper are as follows: How would competition among information providers affect the informational and financial market structure? Would competing information providers sell better (more precise) information to more traders? Does competition always produce a better (in terms of consumer welfare) outcome? In particular, would information providers still be able to appropriate all or part of the informed traders' surplus?

The model considered in this paper is reminiscent of the technology adoption models of Farrell and Saloner (1985) and Katz and Shapiro (1985). Similarly, the proposed model features both strategic complementarity and a spillover effect; spillovers in Katz and Shapiro (1985) are positive, whereas in my model, information revelation through prices leads to negative spillovers. Furthermore, I do not rely on exogenously given demand curves. The preferences of agents for information are indeed endogenous and are derived directly from the preferences for terminal consumption.

First I characterize the information acquisition decision in a market that consists of homogeneous atomistic traders, or, using the terminology of Admati and Pfleiderer (1987), all viable information allocations. There exist three well-defined regimes that are dependent on signal parameters and the decision of information providers regarding the quality of the signal and the number of signals sold. In the first regime (referred to as the 'C regime') signals plays the role of complements. The traders who purchase one signal would necessarily purchase the second signal. In the second regime (later denoted as mixed or the 'M' regime) only a proportion of the agents who purchase the first signal would choose to purchase the second signal as well. In the third regime (S) signals play the role of substitutes. No trader who purchases a single signal will purchase a second signal. Although these regimes can be understood loosely as ones in which signals act as complements (C), substitutes (S) or intermediate (M), the precise role of each signal is a function of both spillover of information through the price system and correlation between signals. Both M and C regimes shrink as signals become more correlated. Such an information allocation structure is different from the one described by Admati and Pfleiderer (1987). This difference is due to direct accounting for the propagation of the information through prices. Thus, the single signal buyer can still benefit from the presence of another signal on the market.

On the information supply side, the presence of competition significantly changes the information selling process. While a monopolistic information provider in Admati and Pfleiderer (1986) limits the number of users in an attempt to control the value of information, competing providers would never choose to do that. However, if signals are uncorrelated, information providers are still able to appropriate the consumer surplus by pricing signals in a way that makes them strategic complements. The price itself can be either higher or lower than the price of the same signal, if offered in a monopolistic setting. This leads to the financial information providers being able to appropriate all of the consumer surplus.

More importantly, the improvement in informational efficiency of the market, associated with the sale of information (as measured by precision of the signal offered for sale), is no higher than in monopolistic settings under the same supply noise and risk aversion parameters.

The reasons the results reported here are so different from those of the extant literature are remarkably simple. First, I take into account the propagation of information through prices. If the signals offered to the market are not correlated, then both the uninformed agents and the agents who buy only one signal learn more about the underlying value of the asset than if the signals were correlated. Second, I let the decision of the agent to purchase one or two signals be endogenous. Information providers here can choose only the number of copies they are selling, but can neither control how many signals are bought by each trader nor discriminate on prices based upon the purchasing of the competitor's signal.

The effects of costly information production are also considered. The presence of an intrinsic limit on the precision of the signal leads to more tacit collusion among information providers. Note that the presence of the cost of information production forces providers to choose their technologies to be as uncorrelated as possible.

The paper is organized as follows. The model is described in detail in Section 2. Section 3 describes viable information allocations. The information providers' optimization problem is analyzed in Section 4. The example of asymmetric equilibria is considered in Section 5. Section 6 concludes the paper, outlining the main results, and suggests topics for future work.

2. The Model

The general structure of my model is similar to those of Grossman and Stiglitz (1980) and Admati and Pfleiderer (1986), and can be described as follows:

2.1. Traders

Assumption 1 (Atomistic CARA traders). *There is a continuum of atomistic traders indexed by* $\nu \in [0, 1]$. *All traders are assumed to exhibit constant absolute risk aversion, i.e., their utility functions are negative exponential.*

Assumption 2 (Homogeneous markets). *All traders have the same absolute risk aversion coefficient* γ.

Assumption 3 (No information resale). *If given the information, traders cannot resell it.*

Thus, each trader is too small to affect prices, and consequently acts as price taker. Assumption 3 can be justified by the high fixed cost necessary to build a distribution network or to establish a reputation à la Benabou and Laroque (1992).

2.2. Investment Opportunities

Assumption 4 (Assets). *There are two assets in the economy, risky and risk-free. The terminal payoff of the latter is* $R = 1$, *whereas the risky asset pays* $\widetilde{F} \sim N(\mathbf{F}, 1)$.

It is convenient to work with normalized variables. Here, we use the normalization $VarF = 1$. If necessary, it can be easily restored by using dimension analysis.

Assumption 5 (Random Supply). *The per capita supply of the risky asset on date 2 is a random variable* $\widetilde{Z} \sim N(0, \sigma_Z^2)$ *which is independent of any information parameter.*

The assumption that noise trading is independent of the information structure in the economy is a simplification that has been adopted in previous noisy rational expectations literature. However, the rise of "day trading" phenomenon can justify this assumption.

2.3. Information Providers

Assumption 6 (Information Providers). *There are two information providers, each endowed with a separate signal about the realization of* $\widetilde{F}$, $S_i = F + \widetilde{\theta}_i$,

$i = 1, 2$, where F is the true realization of $\widetilde{F}$, and the noise in the signals is distributed as

$$\widetilde{\boldsymbol{\theta}} = \begin{pmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{pmatrix}, \ \mathbb{E}\widetilde{\boldsymbol{\theta}} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ Var\widetilde{\boldsymbol{\theta}} = \begin{pmatrix} \sigma_\theta^2 & \rho\sigma_\theta^2 \\ \rho\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix}, \tag{1}$$

where $\rho \in [0, 1]$ is the correlation coefficient between errors of information production technologies 1 and 2.

Obviously, if signals (factors) are perfectly correlated, they are perfect substitutes. However, if correlation is not perfect, an observer of two signals knows more than an individual who buys just one of the signals. The question of combining observations from dependent sources was studied in Winkler (1981), who concluded that the best estimate (in the mean-square sense) is

$$\theta_{12} = \frac{\theta_1(\sigma_{\theta 2}^2 - \rho\sigma_{\theta 1}\sigma_{\theta 2}) + \theta_2(\sigma_{\theta 1}^2 - \rho\sigma_{\theta 1}\sigma_{\theta 2})}{\sigma_{\theta 1}^2 + \sigma_{\theta 2}^2 - 2\rho\sigma_{\theta 1}\sigma_{\theta 2}}, \tag{2}$$

$$Var\theta_{12} = \frac{\sigma_{\theta 1}^2\sigma_{\theta 2}^2\left(1 - \rho^2\right)}{\sigma_{\theta 1}^2 + \sigma_{\theta 2}^2 - 2\rho\sigma_{\theta 1}\sigma_{\theta 2}}, \tag{3}$$

and, in the case of symmetric noise in signals,

$$\begin{aligned} \theta_{12} &= (\theta_1 + \theta_2)/2, \\ Var\theta_{12} &= \sigma_\theta^2 (1 + \rho)/2. \end{aligned}$$

Less-than-perfect correlation between signals might be due not only to a natural divergence of opinions between experts, but also the result of following different forecasting techniques. For example, Frankel and Froot (1990) document the difference between the results produced by the so-called "fundamentalists" and "chartists" in the context of the foreign exchange market.[3]

There are three time periods in the basic model. On date 1, the information is observed by information providers, and is sold to a known number of traders. Trading on the speculative market happens on date 2. On date 3, the riskless asset pays one unit of the consumption good, and the risky asset pays $\widetilde{F}$ units of it.

[3] The example of a "fundamentalists" survey is *Economist Financial Reports*, whereas "chartists" are best represented by *Money Market Survey, Inc.* reports. It is worth noting that there is non-negligible dispersion of opinion between followers of the same forecasting technique; see Frankel and Froot (1990) for details.

For tractability, only the case of duopoly will be considered. However, the extension to the case where $n > 2$ companies compete on information markets is straightforward (although analytically challenging).

In what follows, it is assumed that the information providers indeed observe the signal they are offered for sale. Thus, the problem of information reliability is ruled out. To the author's knowledge, the only paper that considers the problem of information reliability is that of Allen (1990). However, the Allen approach relies heavily on the assumption of independence of the value of information of the number of users. In other words, it is valid only at the limit where the number of information users is small with respect to the total number of traders.

Assumption 7 (Symmetry). *Both information providers have the same information gathering capability, i.e., the signals that they produce have the same variance.*

This structure can be presented as follows: On date 0 one of the providers chooses the variance of his signal. This decision is certified by a trade association, or another body that has the power to regulate the quality of services provided in financial markets. Immediately after that second provider enters the market and chooses the correlation between his technology and that of the first information provider.

Assumption 8. *If the production of signals with different levels of noise leads to the same value of profits, information providers would choose to produce a noisier signal.*

This assumption captures the intuition that the cost of information production increases with the precision of the signal.

Assumption 9 (No Discrimination). *Information providers can limit the number of customers served. They cannot, however, condition the sale on the customer decision concerning purchase of the competitor's signal.*

What this means is that information provider i announces the number of copies of the newsletter he is going to sell (we shall denote this quantity as Λ_i) at a given price. However, he cannot charge a different (discounted) price if the customer purchases his competitor's product. Nor can he contractually prevent his customer from buying the report from competing outlet.

Assumption 10 (Joint Normality and Independence). *The random variables $\widetilde{F}$, $\widetilde{Z}$, $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ are jointly normally distributed. The random variables $\widetilde{F}$, $\widetilde{Z}$, $\widetilde{\theta}_i$ are mutually independent.*

The independence assumption is simplifying, and can be easily relaxed, as long as the random variables are not perfectly correlated.

2.4. Equilibrium and Speculative Market

In the rational expectations equilibrium, both informational and asset markets clear (in a per capita sense) while each trader and information provider maximizes expected utility of final consumption, conditional on all information available to him, including current asset prices. The formal definition can be found in Admati (1985).

Here, I would characterize the linear pricing rule[4] for the market in which there are two sellers of signals, a fraction λ_i of traders buy a single signal θ_i, and a fraction λ_{12} buys both signals, $\Lambda_i = \lambda_i + \lambda_{12}$. It is obvious that, under Symmetry Assumption 7, the number of copies sold by each provider coincides, $\Lambda_1 = \Lambda_2 \equiv \Lambda$. The following proposition describes the equilibrium pricing rule.

Proposition 1. *There exists a unique equilibrium price function*

$$P(\theta_1, \theta_2, Z) = \alpha_1 + \alpha_2 \widetilde{\omega}_\lambda,$$

where α_i *are some constants,*

$$\widetilde{\omega}_\lambda = F + \frac{\widetilde{\theta}_1 + \widetilde{\theta}_2}{2} - p_Z \widetilde{Z}, \tag{4}$$

and p_Z *is the only real solution of*

$$\frac{8p_Z^2\sigma_Z^2\left(\Lambda - \lambda_{12}\right)}{\sigma_\theta^2\left(4p_Z^2\sigma_Z^2 + \left(1-\rho^2\right)\sigma_\theta^2\right)} + \frac{2\lambda_{12}}{\sigma_\theta^2\left(1+\rho\right)} - \frac{\gamma}{p_Z} = 0. \tag{5}$$

The proof of this proposition is given in Appendix A. As in Grossman and Stiglitz (1980) and Admati and Pfleiderer (1986), p_Z is inversely proportional to the informativeness of the price system. The informativeness of the price system is (a) increasing in the number of informed traders, (b) decreasing in traders' risk aversion, (c) increasing in the number of agents buying information, and (d) decreasing in correlation between signals. Note also that there is a strong dependence of the informativeness of the price system on quality of information, σ_θ.

[4] As has become customary in noisy rational expectations literature, I consider only a linear pricing function.

3. Traders' Preferences for Information.

It is well known (see Admati and Pfleiderer (1987)) that, in the economy outlined above, the *ex ante* value of signal that changes traders' information set from $\mathcal{F}_\alpha$ to $\mathcal{F}_\beta$ is given by

$$c_{\alpha\beta} = \frac{1}{2\gamma} \ln \left(\frac{Var(F|\mathcal{F}_\alpha)}{Var(F|\mathcal{F}_\beta)} \right) \tag{6}$$

and it is the maximum that the trader is ready to pay for signal(s). Value of information is an increasing function of precision of the signal. Contrary to popular opinion, it is not necessarily decreasing in the risk aversion coefficient γ, but rather is a complex function of risk aversion.

By employing the projection theorem and results of Proposition 1, one can arrive at the expressions for estimates of an agent's variance (these expressions are given in equation (24) in Appendix A); using these expressions, the valuation of the signal when an agent buys one signal, or both, is

$$c_{0I} = \frac{1}{2\gamma} \ln \left(1 + \frac{\sigma_Z^2 p_Z^2}{\left(\frac{1+\rho}{2}\right) \sigma_\theta^2 \left(1 + \sigma_Z^2 p_Z^2 + \sigma_\theta^2 \left(\frac{1+\rho}{2}\right)\right)} \right), \tag{7}$$

$$c_{0\pi} = \frac{1}{2\gamma} \ln \left(1 + \frac{\sigma_Z^4 p_Z^4}{\left(1 + \sigma_Z^2 p_Z^2 + \sigma_\theta^2 \left(\frac{1+\rho}{2}\right)\right) \left(\sigma_Z^2 p_Z^2 \sigma_\theta^2 + \left(\frac{1-\rho^2}{4}\right) \sigma_\theta^4\right)} \right). \tag{8}$$

I would refer to an agent who is buying a single signal as partially informed, or $\pi-$informed, whereas a buyer of both signals is referred to as fully informed or as an $I-$informed agent. It is easy to show that $c_{0I} \to c_{0\pi}$ as $\rho \to 1$.

The problem of the valuation of the signal is not a trivial one. As can be seen from equation (6), the value of the signal for an individual trader is a function of the informational allocation among other traders. As shown below, the information allocation depends on signals being complements or substitutes. The following is the formal definition adopted from Admati and Pfleiderer (1987):

Definition 1. *Two signals, S_1 and S_2, are complements (substitutes) relative to price system ω_λ if*

$$c_{0I}(S_1, S_2|\omega_\lambda) \geq (\leq) c_{0\pi}(S_1|\omega_\lambda) + c_{0\pi}(S_2|\omega_\lambda)$$

or, for symmetric signals,

$$c_{0I}(S_1, S_2|\omega_\lambda) \geq (\leq) 2c_{0\pi}(S_1|\omega_\lambda).$$

In the information market with two independent information providers who price their signals according to equations (7) and (8), there exists the possibility for traders to endogenously reach allocations in which part of the market is $\pi-$informed, and part is fully informed. Thus, signals in this region of parameters are neither complements nor substitutes. The following lemma specifies the conditions under which buying a double signal is equivalent to buying one signal.

Lemma 1. *In homogeneous markets, traders are indifferent when it comes to buying a single signal or a combined signal if and only if the informativeness of the price system is equal to*

$$(p_Z^*)^2 = \frac{(1-\rho)\,\sigma_\theta^2\left[1-\rho+\sigma_\theta^2(1-\rho^2)+\phi\right]}{4\sigma_Z^2\left(1+\rho+2\rho\sigma_\theta^2\right)}, \tag{9}$$

where $\phi = \sqrt{5+4\sigma_\theta^2\,(1+\rho)^3+\sigma_\theta^4\,(1+\rho)^4+\rho\,(6+5\rho)}.$

P roof. Straightforward, by equating $c_{0I} = 2c_{0\pi}$ and employing equations (7) and (8). ■

Using the result of this lemma, it is possible to explicitly define viable information allocations. Obviously, the condition $c_{0I} = 2c_{0\pi}$ can be satisfied only in some region of the parameters where, effectively, traders reallocate information themselves. The mechanics of such a reallocation is not explicitly modeled in this paper. It is assumed, instead, that there exist frictionless secondary markets for any unopened newsletters, where the exchange and/or trade of newsletters can be carried out. The following proposition (proven in Appendix 2) describes the resulting structure of the demand side of the information market.

Proposition 2. *Ceteris paribus, if*

$$\sigma_Z < \sigma_Z^* \equiv \frac{\Lambda\left(3+\rho+\sigma_\theta^2\,(1+\rho)^2-\phi\right)}{\gamma\sigma_\theta(1+\rho)}\sqrt{\frac{\left(1-\rho+\sigma_\theta^2\,(1-\rho^2)+\phi\right)}{\left(1+\rho+2\rho\sigma_\theta^2\right)(1-\rho)}}, \tag{10}$$

then signals are substitutes, and the market consists of 2Λ *partially informed and* $(1-\ 2\Lambda)$ *uninformed traders. If*

$$\sigma_Z \geq \sigma_Z^{**} \equiv \frac{\Lambda\sqrt{1-\rho}}{\gamma\sigma_\theta(1+\rho)}\sqrt{\frac{1-\rho+\sigma_\theta^2\,(1-\rho^2)+\phi}{1+\rho+2\rho\sigma_\theta^2}}, \tag{11}$$

then signals are complements, and the market consists of Λ *fully informed and* $(1-\Lambda)$ *uninformed traders. Otherwise, if* $\sigma_Z \in (\sigma_Z^*, \sigma_Z^{**})$, *a non-zero fraction,* λ, *of the market would choose to buy a single signal, whereas* $(\Lambda - \lambda)$ *would buy both signals, and the remainder would remain uninformed. Similarly, if* $\rho < 1$,

$$\Lambda > \Lambda^* \equiv \min\left(1, \frac{\gamma\sigma_\theta\sigma_Z\left(2+\rho+\rho^2+\sigma_\theta^2(1+\rho)^2+\phi\right)}{\left(1-\rho+\sigma_\theta^2(1-\rho^2)+\phi\right)^{3/2}}\sqrt{\frac{1+\rho+2\rho\sigma_\theta^2}{1-\rho}}\right), \tag{12}$$

then 2Λ *traders would buy just one signal; if*

$$\Lambda < \Lambda^{**} \equiv \min\left(1, \frac{\gamma\sigma_\theta\sigma_Z(1+\rho)}{\sqrt{1-\rho}}\sqrt{\frac{1+\rho+2\rho\sigma_\theta^2}{1-\rho+\sigma_\theta^2(1-\rho^2)+\phi}}\right) \leq \Lambda^* \tag{13}$$

then Λ *traders would buy both signals and become fully informed. In the region of parameters* $\Lambda \in (\Lambda^{**}, \Lambda^*)$ *a mixed equilibrium is observed.*

As correlation between signals increases, the propagation of the information through the price system decreases. Both σ_Z^* and σ_Z^{**} decrease proportionally to $\sqrt{1-\rho}$ as ρ achieves 1. However, the difference between these two functions (which determines the M regime width) decreases as $(1-\rho)$, much faster than σ_Z^* and σ_Z^{**}. Both σ_Z^* and σ_Z^{**} increase with Λ and decrease with the risk aversion coefficient, γ. An example of σ_Z^* and σ_Z^{**} is shown in Figure 1. If the signals are very precise and supply noise is high, or if signals are not very valuable, but supply noise is small, then traders choose to buy only one signal. In the former case, the agents do not want a second signal because the price of risky asset is determined mostly by supply noise. In the latter case, information easily propagates through the price system, causing the value of information to deteriorate.

The second half of Proposition 2 shows that if providers choose to sell a small number of copies, then investors buy two signals. However, if there are many informed traders in the market and the price system is informative enough, then the propagation of information through the price system can make the second signal redundant. Note that the behavior of Λ^* as a function of σ_θ is different for slightly correlated and uncorrelated signals. If signals are not correlated at all, $\lim_{\sigma_\theta\to\infty}\Lambda^* = \min\left(2^{-1/2}\gamma\sigma_Z,\ 1\right)$, and if $\rho > 0$, $\lim_{\sigma_\theta\to\infty}\Lambda^* = 1$. If the supply noise or risk aversion of traders is high, it is possible that $\Lambda^* = 1$ and the S regime for some values of signal precision does not exist. If, in addition, $\Lambda^{**} = 1$, the M regime ceases to exist as well, as all agents would choose to become fully informed. Such a situation is shown in Figure 2. It is worth mentioning that some regions on this phase diagram are virtually unattainable.

Adding a new signal that is not perfectly correlated with the existing one would reduce uncertainty in the economy either directly (for the agents who buy

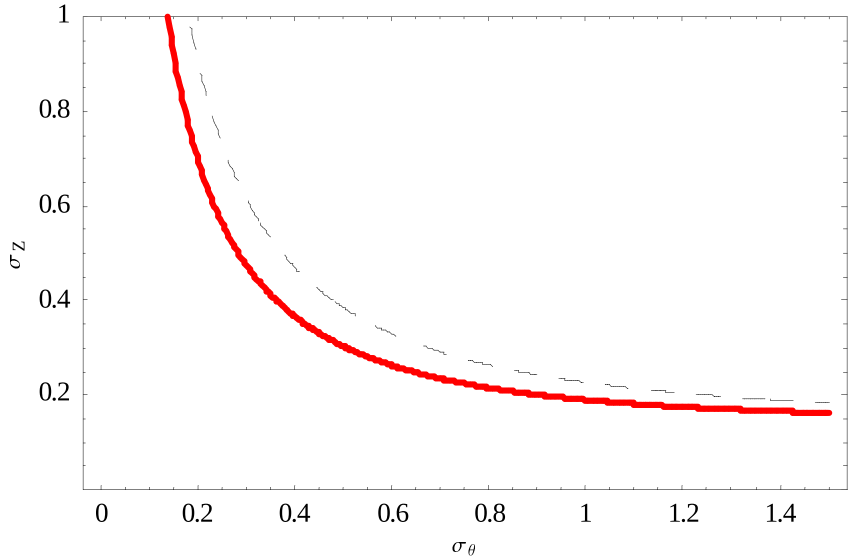

Figure 1: Viable information allocations as functions of noise in signal σ_θ and risky asset supply σ_Z. The boundaries between S and M regimes σ_Z^* (bold line), and C and M regimes σ_Z^{**} (dashed line) are drawn for $\Lambda = 0.1$, $\gamma = 1$, $\rho = 0$.

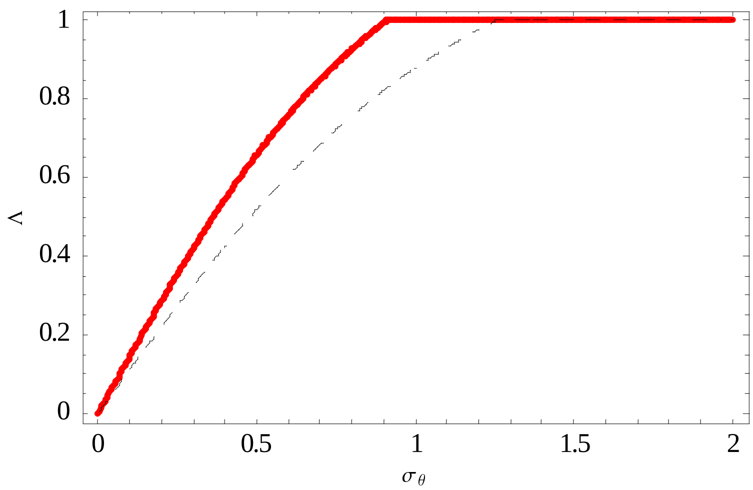

Figure 2: Viable information allocations for a duopoly information market as a function of the number of copies offered for sale Λ, and signal precision σ_θ. Λ^* is depicted by the bold line, and Λ^{**} by the dashed line. The parameters used are as follows: $\rho = 0$, $\sigma_Z = 0.4, \gamma = 5$.

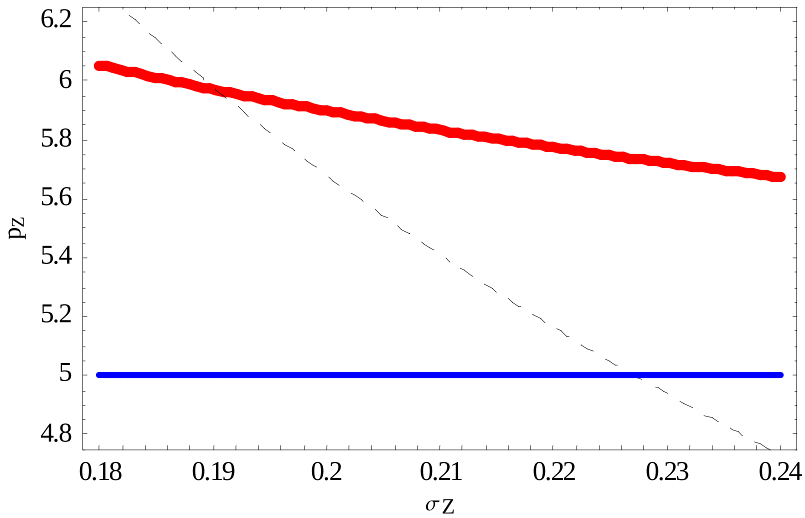

Figure 3: Inverse informativeness of the price system, p_Z, as a function of supply noise variance. The thin horizontal line corresponds to the C regime, the dashed line corresponds to the M regime, and the bold line corresponds to the S regime. The crossover points between regimes correspond to the switch to a "more informative" equilibrium. The parameters are $\Lambda = 0.1$, $\gamma = 1$, $\rho = 0$.

the new signal) or indirectly (through the price system, for agents who do not buy the signal.) The behavior becomes entirely different if signals are perfectly correlated, as, in this case, the value of any additional signal is zero, so everyone buys only one signal.

It is interesting to compare the informativeness of the price system in these regimes. Figure 3 shows the behavior of p_Z as a function of supply noise. At σ_Z^*, there is a transition from the price system that is shown by the bold line to the more informative one (dashed line). At σ_Z^{**}, a similar kink happens again, from the price system that corresponds to a "mixed" market to the price system that corresponds to a "C" market (thin horizontal line.) However, the kink at σ_Z^{**} is caused by the fact that informativeness of the price system cannot be further increased by an increase in the number of traders who buy both signals (as the capacity is reached). Thus, the transition between regimes corresponds to the choice by agents of the more informative price system that benefits both partially informed and uninformed agents.

It is important to note that, in this section, we assume that the signals are priced at their higher value. If the signals are priced lower than the price defined by equations (7) and (8), the traders will be more inclined to buy both signals. As a result, the region where signals are substitutes will shrink.

4. Information Production Optimization.

As shown above, the structure of traders' preferences for information is quite complex. Obviously, information providers' optimization within such a preference structure is a difficult task. For simplicity, the discussion below is limited to two cases. First, a benchmark case of perfectly correlated signals is considered. Information duopoly with indistinguishable signals is easily comparable with the monopolistic setting results of Admati and Pfleiderer (1986). In the other extreme, the case of uncorrelated signals is also considered. The generalization of the results reported below to the case of arbitrary $\rho \in (0,1)$ is straightforward.

4.1. Perfectly Correlated Signals.

We now consider the market with two information providers who offer the market indistinguishable signals produced at zero cost. The price p, at which an information provider can sell his signal, is bounded above by

$$c_{0I} = \ln\left(1 + \frac{1}{\sigma_\theta^2 + \left(1+\sigma_\theta^{-2}\right)\left(\frac{\Lambda_1+\Lambda_2}{\gamma\sigma_Z}\right)^2}\right),$$

which happens to be the monopoly price of the signal (if $\Lambda_2 = 0$). The first question one should ask is, can providers compete by choosing optimally both the quantity and the quality of the information he wishes to sell? Such a provider faces the profit maximization problem

$$\max_{0<\Lambda_i<1/2,\sigma_\theta} \Lambda_i \ln\left(1 + \frac{1}{\sigma_\theta^2 + \left(1+\sigma_\theta^{-2}\right)\left(\frac{\Lambda_1+\Lambda_2}{\gamma\sigma_Z}\right)^2}\right). \tag{14}$$

This problem is similar to the one considered in a monopolistic setting by Admati and Pfleiderer (1986). However, it appears that the results are quite different.

Lemma 2. *In the economy with perfectly correlated signals, there exists no Cournot-type equilibrium.*

P roof. To show that there is no sustainable Cournot-type equilibrium is equivalent to showing that there exists a unique solution $(\Lambda, \sigma_\theta^2)$ to problem (14), where both information providers sell information to the whole market ($\Lambda_1 = \Lambda_2 = \frac{1}{2}$), and variance is set to be $\sigma_\theta^2 = (\gamma\sigma_Z)^{-1}$.

It is easy to show that the choice of signal variance $\sigma_\theta^2 = \frac{\Lambda_1+\Lambda_2}{\gamma\sigma_Z}$ makes (14) equivalent to the problem

$$\max_{0<\Lambda_i<1/2} \Lambda_i \ln\left(1 + \frac{1}{2\left(\frac{\Lambda_1+\Lambda_2}{\gamma\sigma_Z}\right) + \left(\frac{\Lambda_1+\Lambda_2}{\gamma\sigma_Z}\right)^2}\right).$$

Obviously, the optimal choice is symmetric, $\Lambda_1 = \Lambda_2 = \gamma\sigma_Z x/2$. First order conditions can be written as

$$\frac{\partial\Pi}{\partial\Lambda} = \ln\left(1 + \frac{1}{2x + x^2}\right) - \frac{1}{(2+x)(x+1)}.$$

It is easy to see that, at zero, this expression goes to infinity. At $x \gg 1$, this expression is approximately equal to $x^{-3} + o(x^{-4}) > 0$, and approaches zero as $x \to \infty$. At the same time, the second order conditions yields $\frac{\partial^2\Pi}{\partial\Lambda^2} > 0$. Thus, the constrained optimal solution of equation (14) is given by $\Lambda_1 = \Lambda_2 = \frac{1}{2}$, $\sigma_\theta^2 = (\gamma\sigma_Z)^{-1}$. ■

Lemma 2 shows the important difference between duopoly and monopoly cases. While a monopolist tries to limit the number of information users, competing information providers would never do so. Thus, the providers of indistinguishable products face a market that is limited in size. In other words, information providers face Bertrand competition which drives the price of the signal to marginal cost. This result is outlined in the following proposition:

Proposition 3. *In information markets with perfectly correlated signals, information providers compete in prices. The price they are able to charge is equal to the marginal cost of production.*

The driving force behind this result is the absence of commitment on the part of information providers to limit the number of copies of information sold. If such a commitment were possible (either through regulation or increasing marginal costs) providers would be able to capture a larger part of the consumer surplus.

4.2. Uncorrelated Signals.

Operating in information markets with uncorrelated signals adds further complexity to the optimization problem of the information provider. To keep notations tractable, I will consider only the case where signals are completely uncorrelated, $\rho = 0$. The generalization of the discussion below to the case of arbitrary ρ is straightforward.

The main results of this section are described in the following proposition and corollaries.

Proposition 4. *Competing information providers, whose signals are uncorrelated, do not limit the number of market participants. They produce the signal with precision* $\sigma_\theta^2 = \frac{2}{\gamma\sigma_Z}$. *While pricing their signal, they do it in a way that makes the signals strategic complements and induces traders to purchase both signals. The charge for their information is*

$$\frac{1}{4\gamma}\ln\left(1+\frac{\gamma^2\sigma_Z^2}{1+2\gamma\sigma_Z}\right).$$

Corollary 1. *The quality of the signal offered for sale by an information provider under competition is no higher than that offered by the same provider in monopolistic settings.*

Corollary 2. *Competing information providers extract all of the consumer surplus.*

As shown above, traders' preferences for information vary according to the total information allocation in the economy. Not surprisingly, S, M, and C regimes must be considered separately. The following lemma is proved in the Appendix 3:

Lemma 3. *In the absence of a credible commitment mechanism, a rational information provider does not limit the number of information users.*

This lemma is analogous to Lemma 2 in the case of perfectly correlated signals. The intuition behind this result is straightforward: in the case of uncorrelated signals, an increase in the number of buyers causes the value of information to deteriorate even further than for perfectly correlated signals. Spillovers of additional information occur through the price system, thus increasing the incentive for the information provider to sell even more copies.

Similarly to the case of perfectly correlated signals, it amounts to the non-existence of Cournot-type equilibria. However, competition in prices of differentiated signals results in pricing the information in a way that makes the signals strategic complements.

In the C regime, by definition, signals act as complements and $c_{0I} > 2c_{0\pi}$. Thus, the price the information providers can charge is actually higher than the sum of individual signal values. It is easy to see that the natural behavior for an information provider is to charge $p = c_{0I}(\Lambda = 1)/2 > c_{0\pi}$.

The intuition behind this result is clearer when we look at the region of signal and supply noise volatilities that satisfy condition $\Lambda^{**} = 1$. Using equation (13), it is easy to show that, for the case $\rho = 0$,

$$\gamma^2 \sigma_Z^2 \sigma_\theta^2 \geq 1 + \sigma_\theta^2 + \sqrt{5 + 4\sigma_\theta^2 + \sigma_\theta^4}, \tag{15}$$

or, substituting first order conditions, $\sigma_\theta^2 = 2\left(\gamma\sigma_Z\right)^{-1}$,

$$2\gamma^2\sigma_Z^2 - 2 - \gamma\sigma_Z - \sqrt{4 + 8\gamma\sigma_Z + 5\gamma^2\sigma_Z^2} \geq 0,$$

which leads to $\gamma\sigma_Z \geq 1 + \sqrt{2}$. Thus, the trade-off here is between the possibility of hiding information (if the supply noise is high enough) and the high precision of the signal (which is increasing in the supply noise). Given an exogenous level of supply noise, information providers choose to produce signals that are precise enough to induce traders to purchase both signals, but not more than necessary in order to limit the spillovers of information through prices.

Thus, if $\gamma\sigma_Z \geq 1 + \sqrt{2}$, then information providers choose to produce the signals that have a precision

$$\sigma_\theta^2 = 2\left(\gamma\sigma_Z\right)^{-1} \tag{16}$$

and price it at $p = c_{0I}(\Lambda = 1)/2 > c_{0\pi}$. While doing so, each provider takes into account the effect of other provider's sales. This solution can be interpreted as tacit collusion.

Observe that, if the signals are substitutes, $\Lambda > \Lambda^*$, by definition $2c_{0\pi} \geq c_{0I} > c_{0\pi}$. Imagine that the provider of signal 1 would charge the price that is smaller than $c_{0\pi}$, $p = c_{0\pi} - \varepsilon$. To prevent the first provider from capturing the whole market, the second provider should drive the price down as well. Note that an asymmetric equilibrium is not an option in the absence of a commitment mechanism, as the information provider profit is always decreasing in the other provider's sales and increasing in his own. Such competition would lead to the point where price is equal to $c_{0I}(\Lambda = 1)/2$. At this point, all rational traders would purchase both signals. The optimal noise level is again given by Equation (16).

As a result, for all values of supply noise, the information providers price the signal in such a way that induces traders to purchase both signals. However, the nature of such a decision is different, dependent on the level of supply noise. If noise or risk aversion is small, $\gamma\sigma_Z < 1+\sqrt{2}$, the decision of information providers is driven by competition. Should one of the providers exit the market, the price of the signal would necessarily rise. Conversely, if the supply noise is high, an information provider would benefit from the presence of a rival on the market. The price charged is higher than the price of an individual signal.

As the number of traders served is always higher in a duopoly case than in a monopoly one, the precision of a cumulative signal produced by duopolists is always lower than one produced by monopolists. Thus, the amount of information available to market participants is lower in a duopoly case than in monopoly one. As far as welfare goes, it seem to be difficult to arrive at any definite conclusion because of the noise trading involved.

If the production of information is costly, information providers choose the correlation between signals to be as low possible, thus, to increase $\sigma_\theta^2 = \frac{2}{(1+\rho)\gamma\sigma_Z}$ and to cut the "production cost". The incentive to be different works in the opposite direction to the incentive to herd, which has been considered in recent literature.

In Appendix 4, the effects of costly information gathering is considered. It is shown that the results of this section are robust to changes in the information production cost function.

5. Asymmetric information allocation.

It is worth mentioning that we restrict our attention to symmetric equilibrium. Katz and Shapiro (1985) show that asymmetric equilibrium in duopoly settings is possible. Direct calculations show that, without imposing constraints on an individual provider's share of the market, in the settings of this paper, such an equilibrium does not exist. However, it might appear if a more sophisticated market structure is imposed.

The results above are derived under the assumption that information providers cannot credibly commit themselves to serving only a fraction of the market. In this section, the polar case is considered. I would assume that one of the information providers commits himself to serving the whole market, $\Lambda_1 = 1$, and to selling the signal at its fair value $c_{0\pi}(1, \Lambda_2)$. The mechanism of such a commitment is not specified. One can think of regulatory pressures associated with a requirement to maintain a uniform price throughout the market. An example of such regulatory pressures can be easily found in emerging markets, where local companies are heavily regulated but can serve both domestic and foreign clients, whereas foreign financial service companies are constrained to deal with only foreigners. It is also assumed that regulators determine the precision of the signal to be served. A more compelling interpretation is to associate a publicly available signal with the public disclosure of information. In such a case, the second signal is associated with private information.

For the sake of simplicity, only the case of uncorrelated signals is considered, $\rho = 0$. Let us assume that, through the use of unspecified means, a first provider was able to capture the whole market, $\Lambda_1 = 1$. The question we are asking here is: What are the strategies available to the second information provider?

First, it is possible to show that the pricing rule in this case is governed by the following lemma:

Lemma 4. *For the case of asymmetric information allocation, the equilibrium pricing rule is given by*

$$P = \alpha_0 + \alpha_1 \overline{\omega}_{\{\lambda\}},$$

where α_i are some constants and

$$\overline{\omega}_{\{\lambda\}} = S_1 \frac{\gamma^2 \sigma_Z^2 \sigma_\theta^2 + \Lambda_2^2}{(\gamma^2 \sigma_Z^2 \sigma_\theta^2 + \Lambda_2)\sigma_\theta^2} + S_2 \frac{\Lambda_2}{\sigma_\theta^2} - \gamma Z. \tag{17}$$

The proof of this lemma is based on a straightforward application of the projection theorem (22), and is omitted. The situation described here boils down to the fact that, while the first information provider sells to the whole market, the second provider sells to a (yet undetermined) number of customers $\Lambda_2 \in [0,1]$. As for the pricing rule, it is noteworthy that (a) it becomes symmetric as Λ_2 approaches 1, and (b) the contribution of the first information provider to the pricing rule is actually non-monotonic, with a minimum at $\Lambda_2 = 0.5$. Note that the problem considered here is actually isomorphic to the monopolistic information provider, if the prior probability distribution depends on signal S_1.

The value of both signals is a decreasing function of Λ_2 :

$$\begin{aligned}
c_{0\pi}(\Lambda_1 &= 1, \Lambda_2) = \frac{1}{2\gamma} \ln \left(1 + \frac{z_1^4 \left(z_1^2 + \Lambda_2 \right)}{z} \right), \\
c_{0I}(\Lambda_1 &= 1, \Lambda_2) = \frac{1}{2\gamma} \ln \left(1 + \frac{z_1^2 \left(2 z_1^4 + 2\Lambda_2^2 + z_1^2 \left(1 + \Lambda_2 \right)^2 \right)}{z} \right), \\
z &= z_1^6 + 2\Lambda_2^4 (2 + \sigma_\theta^2) + z_1^4 \left(1 + \sigma_\theta^2 \right) \left(1 + \Lambda_2 \right)^2 \\
&\quad + z_1^2 \Lambda_2^2 \left(4 + 4\Lambda_2 + \sigma_\theta^2 \left(3 + 2\Lambda_2 \right) \right), \\
z_1 &= \gamma \sigma_Z \sigma_\theta.
\end{aligned}$$

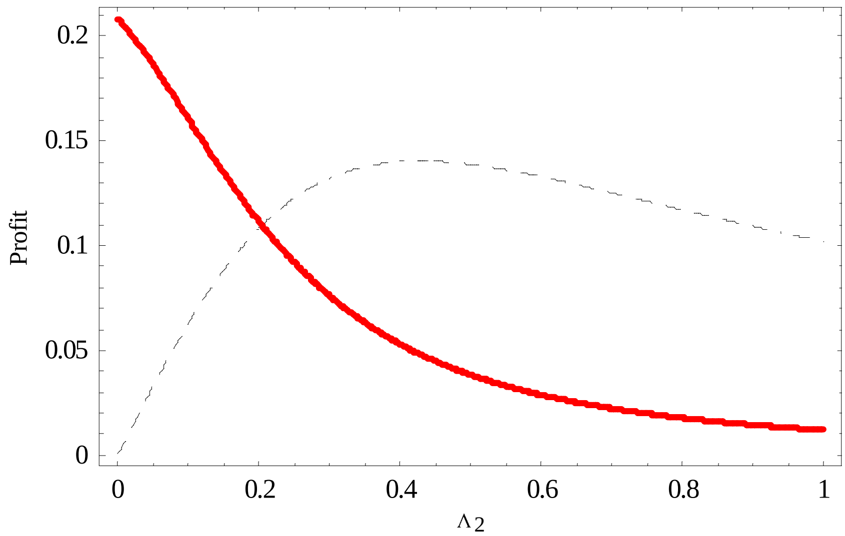

Figure 4: Profit of information producers one (solid line) and two (dashed line) for $\gamma = 1$, $\sigma_Z = \frac{1}{4}$, $\sigma_\theta = 2$.

Correspondingly, profits for information providers are

$$\Pi_1 \equiv c_{0\pi}(\Lambda_1 = 1, \Lambda_2), \tag{18a}$$

$$\Pi_2 = \frac{\Lambda_2}{2\gamma} \ln\left(1 + \frac{1}{1 + \sigma_\theta^2 + \left(\frac{\Lambda_2}{\gamma\sigma_Z}\right)^2 \left(2\sigma_\theta^{-2} + 1\right)}\right). \tag{18b}$$

It is easy to see that equation (18b) actually describes the profit of the monopolistic information provider of Admati and Pfleiderer (1986). Thus, there exists some region of parameters in which there exists an optimal $\Lambda_2^* < 1$. An example of such an equilibrium is shown in Figure 4. It is worth noting that the profit of the second information provider can actually exceed the profit of the first provider.

It is also interesting to note that the public signal can be interpreted as the public disclosure of information regarding the future prospects of the firm, and the restricted signal can be understood as private information sold to the newsletter buyers or customers of brokerage services. Equation (18a) can be interpreted as the welfare gains society gets from the presence of the public disclosure. If so, the solid line in Figure 4 shows that the value of public disclosure rapidly deteriorates as the number of informed traders increases. At the same time, the value of a private signal exceeds the value of a public disclosure. In other words, buyers of a private signal disproportionately benefit from the presence of a public signal as they are able to combine optimally both private and public signals to their informational advantage. Thus, the deterioration of the value of publicly available information should be considered as one of the costs of allowing informed trading on the market.

6. Conclusion

The results presented in this paper show that competition and diversity among financial information providers dramatically changes the nature of markets for information. By allowing signals to be less than perfectly correlated, information providers create differentiation among signals. The price they are able to charge is determined by the value of all the signals they sell. It can be either higher or lower than the price of the same signal produced in the monopolistic setting. More importantly, if the signals are uncorrelated, competition does not lead to any improvement on the part of the traders. As in the monopoly case considered by Admati and Pfleiderer, information providers are able to appropriate all of the consumer surplus. The quality (precision) of the signal supplied is no better than the one a monopolist would choose to produce, under the same supply noise and risk aversion parameters.

Finally, it is shown that, if one signal is revealed to the whole market (e.g., public disclosure), and a second one is revealed to only a subset of investors, it is the latter group who benefit disproportionately from public disclosure.

The important normative question outlined is as follows: Does the diversity of information providers create any real value for information consumers? In our model, the answer is negative. Letting information providers choose both the correlation between their signals and their variance leads to no improvement on the part of traders.

Graham and Harvey (1996) and Jaffe and Mahoney (1999), amongst others, have documented the poor performance of individual newsletters. On the other hand, Kumar (1999) and Barber, Lehavy, McNicols, and Trueman (1998) showed that a consensus forecast can indeed provide valuable information. Investment strategy based on consensus forecast outperforms various benchmarks. This result is robust to delay, size, book-to-market effects, etc. The findings of this paper seem to be in agreement with these results. In the context of our model, poor performance of an individual newsletter corresponds to high variance of individual signals. However, a combined signal can still provide decent information to the traders. It can also explain the rise of services that are selling consensus forecasts either by newsletters (Hulbert Digest) or by brokerage analysts (I/B/E/S, First Call, Zacks, etc.)

It would be interesting to investigate whether there exists a commitment mechanism that is credible enough to limit information selling. Such a commitment can be realized by the information provider holding a certain number of shares of the risky asset. It would also be interesting to investigate trading strategies of information providers in a heterogeneous multi-asset markets. These topics are left for future research.

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A. Proof of Proposition 1

It is easy to show that, for the agents with CARA preferences, the demand for the risky security is given by

$$X_\alpha = \frac{E(F|\mathcal{F}_\alpha) - P}{\gamma \; var(F|\mathcal{F}_\alpha)}, \tag{19}$$

$\alpha = I, \pi, U$. Let us assume (this assumption has to be verified in equilibrium) that the informational content of prices is equivalent to the informational content of the variable

$$\widetilde{\omega}_\lambda = F + (\widetilde{\theta}_1 + \widetilde{\theta}_2)/2 - p_Z \widetilde{Z}, \tag{20}$$

where p_Z is a constant, independent of $(\widetilde{\theta}_i, \widetilde{Z})$, yet to be determined. The information set of each agent is as follows:

$$\mathcal{F}_U = \{\omega_\lambda\}, \mathcal{F}_{\pi i} = \{\omega_\lambda, \theta_i\}, \mathcal{F}_I = \{\omega_\lambda, \theta_1, \theta_2\}. \tag{21}$$

Using the projection theorem

$$E(\mu_0|\boldsymbol{\mu}_1) = E(\mu_0) + Cov[\mu_0, \boldsymbol{\mu}_1] \, Var^{-1}[\mu_0, \boldsymbol{\mu}_1] (\boldsymbol{\mu}_1 - E\boldsymbol{\mu}_1), \tag{22a}$$

$$\begin{aligned} Var(\mu_0|\boldsymbol{\mu}_1) = {} & Var(\mu_0) \\ & - Cov[\mu_0, \boldsymbol{\mu}_1] \, Var^{-1}[\mu_0, \boldsymbol{\mu}_1] \, Cov^{\intercal}[\mu_0, \boldsymbol{\mu}_1], \end{aligned} \tag{22b}$$

where $\boldsymbol{\mu}^{\intercal} = (\mu_0, \; \mu_1, \; ..., \; \mu_N) \equiv (\mu_0, \boldsymbol{\mu}_1)$ is the vector of jointly distributed normal variables, we can write:

$$E(F|\omega_\lambda) = F + \frac{(\omega_\lambda - F)}{1 + \frac{1+\rho}{2}\sigma_\theta^2 + p_Z^2 \sigma_Z^2}, \tag{23a}$$

$$E(F|\omega_\lambda, \theta_i) = \frac{\left(\frac{1-\rho^2}{4}\right) F\sigma_\theta^4 + p_Z^2 \sigma_Z^2 \left(F\sigma_\theta^2 + \theta_i\right) + \omega_\lambda \sigma_\theta^2 \left(\frac{1-\rho}{2}\right)}{\sigma_\theta^2 \left(\frac{1-\rho}{2}\right)\left(1 + \sigma_\theta^2\left(\frac{1+\rho}{2}\right)\right) + p_Z^2 \sigma_Z^2 \left(1 + \sigma_\theta^2\right)}, \tag{23b}$$

$$E(F|\omega_\lambda, \theta_1, \theta_2) = \frac{F(1+\rho)\sigma_\theta^2 + (\theta_1 + \theta_2)}{2 + (1+\rho)\sigma_\theta^2}, \tag{23c}$$

$$Var(F|\omega_\lambda) = \frac{\frac{1+\rho}{2}\sigma_\theta^2 + p_Z^2 \sigma_Z^2}{1 + \frac{1+\rho}{2}\sigma_\theta^2 + p_Z^2 \sigma_Z^2}, \tag{24a}$$

$$Var(F|\omega_\lambda, \theta_i) = \frac{\left(\frac{1-\rho^2}{4}\right)\sigma_\theta^4 + p_Z^2 \sigma_Z^2 \sigma_\theta^2}{\sigma_\theta^2 \left(\frac{1-\rho}{2}\right)\left(1 + \sigma_\theta^2\left(\frac{1+\rho}{2}\right)\right) + p_Z^2 \sigma_Z^2 \left(1 + \sigma_\theta^2\right)}, \tag{24b}$$

$$Var(F|\omega_\lambda, \theta_1, \theta_2) = \frac{(1+\rho)\sigma_\theta^2}{2 + (1+\rho)\sigma_\theta^2}. \tag{24c}$$

Note that the variance of a π-agent approaches the variance of a fully informed agent as signals become more and more correlated. Employing the market clearing condition,

$$\begin{aligned} \lambda_1 X_{\pi 1} + \lambda_2 X_{\pi 2} + \lambda_{12} X_I + (1 - \lambda_1 - \lambda_2 - \lambda_{12}) X_U &= Z, \\ s.t. \lambda_i + \lambda_{12} &= \Lambda_i, \end{aligned} \tag{25}$$

and equations (19, 23, 24), one can write the equation for p_Z as

$$\frac{8 p_Z^2 \sigma_Z^2 (\Lambda - \lambda_{12})}{\sigma_\theta^2 \left(4 p_Z^2 \sigma_Z^2 + (1 - \rho^2) \sigma_\theta^2\right)} + \frac{2 \lambda_{12}}{\sigma_\theta^2 (1 + \rho)} - \frac{\gamma}{p_Z} = 0 \tag{26}$$

There exists a unique real root of this cubic equation, which can be solved analytically. An example of such a solution for the case $\lambda_{12} = 0$ is given in Appendix 3. Using equation (23) it is easy to show that the price relationship (4) is indeed true.■

2. Proof of Proposition 2

Once the number of copies for sale is announced, traders would choose to buy a second signal up to the point where the value of two signals is equal to the sum of the values of separate signals. It is possible that it would never happen, and that the second signal is therefore redundant ($\lambda_{12} = 0$). This happens if the signal is very precise and/or the supply noise is small. It might also happen that the price system is very noisy, and that it pays to buy both signals. This corresponds to $\lambda_{12} = \Lambda$.

From Lemma 1, the condition of equality of $2c_{0\pi} = c_{0I}$ is equivalent to

$$(p_Z^*)^2 = \frac{(1 - \rho) \sigma_\theta^2 \left[1 - \rho + \sigma_\theta^2 (1 - \rho^2) + \phi\right]}{4 \sigma_Z^2 \left(1 + \rho + 2 \rho \sigma_\theta^2\right)}. \tag{27}$$

To find the boundary between the mixed regime and the substitutes regime, one should introduce (27) into equation (26) along with the condition $\lambda_{12} = 0$. Simple manipulations lead to equation (10). Similarly, the introduction of (27) into equation (26), along with the condition $\lambda_{12} = \Lambda$, leads to equation (11).■

3. Proof of Lemma 3

If signals are complements, $\Lambda < \Lambda^{**}$, traders buy both signals, and the information provider's problem is

$$\max_{0<\Lambda_i<\Lambda^{**},\sigma_\theta} \Lambda_i \ln\left(1+\frac{2}{\sigma_\theta^2+4\left(\frac{1}{2}+\sigma_\theta^{-2}\right)\left(\frac{\Lambda_i}{\gamma\sigma_Z}\right)^2}\right).$$

If given a choice of signal noise, the information provider would choose $\sigma_\theta^2 = \frac{2\Lambda_1}{\gamma\sigma_Z}$, and the optimization problem would collapse into one similar to the monopolistic optimization of Admati and Pfleiderer (1986),

$$\max_{0<\Lambda_i<\Lambda^{**}} \Lambda_i \ln\left(1+\frac{\gamma\sigma_Z}{\Lambda_i\left(2+\frac{\Lambda_i}{\gamma\sigma_Z}\right)}\right).$$

It is easy to show that such a problem has a unique unconstrained optimum. Moreover, at small Λ, the first order condition diverges as $\ln\Lambda$, whereas, as Λ approaches infinity, the first order condition converges to zero as $-\left(\frac{\gamma\sigma_Z}{\Lambda}\right)^2$. Thus, if at Λ^{**} first order conditions deliver positive results, there exists no optimum at $\Lambda\in[0,\Lambda^{**}]$. By introducing the root of equation $\Lambda=\Lambda^{**}$ into the first order conditions, one gets $\ln 2+\sqrt{2}-2>0$. Thus, the optimum is outside the interval $\Lambda_i\in[0,\Lambda^{**})$.

However, if $\gamma\sigma_Z \geq 1+\sqrt{2}$, then the signals are complements for any $\Lambda_i\in[0,1]$. In turn, this means that information providers will not limit the number of copies offered for sale as long as supply noise is high enough.

If $\gamma\sigma_Z < 1+\sqrt{2}$, information providers may find themselves in an M market regime, $\Lambda\in[\Lambda^{**},\Lambda^{*}]$. Their profits can be expressed as

$$\Pi = \frac{\Lambda}{4\gamma}\ln\left(1+\frac{\sigma_Z^2 p_Z^2}{\left(\frac{1+\rho}{2}\right)\sigma_\theta^2\left(1+\sigma_Z^2p_Z^2+\sigma_\theta^2\left(\frac{1+\rho}{2}\right)\right)}\right), \tag{28}$$

where $p_Z = p_Z^*$ is given by equation (9). The expression under the logarithm is independent of Λ, and profit increases linearly with Λ up to Λ^*. Thus, the rational information provider chooses to sell to a number of customers that is so high that none of them chooses to buy a second signal. If $\Lambda^*\geq 1$, the optimization involves finding the optimal precision of the signal $\sigma_\theta\in(\sigma_\theta^*,\sigma_\theta^{**})$ of the expression under the logarithm. The first order conditions yield $\sigma_\theta=\sigma_\theta^*$. Thus, if $\gamma\sigma_Z<1+\sqrt{2}$, the information providers choose their precision and number of copies sold in a way that never leads them to be in an M regime. In other words, if equilibrium exists, it does exist somewhere on $[\Lambda^*, \infty)$. This can be seen in Figure 3. The price system is more informative in an M regime than when signals are substitutes (in an S regime). Equivalently, the spillover of information through the price system suppresses the value of the signal and profits.

Let us consider optimization in the case where signals are substitutes. Each information provider maximizes

$$\Pi_i = \frac{\Lambda_i}{2\gamma} \ln \left(1 + \frac{\sigma_Z^4 p_Z^4}{\left(1 + \sigma_Z^2 p_Z^2 + \sigma_\theta^2 \left(\frac{1+\rho}{2}\right)\right) \left(\sigma_Z^2 p_Z^2 \sigma_\theta^2 + \left(\frac{1-\rho^2}{4}\right) \sigma_\theta^4\right)} \right). \tag{29}$$

Using equation (26), this can be written as

$$\Pi_i = \frac{\Lambda_i}{2\gamma} \ln \left(1 + \frac{\gamma \sigma_Z^2 p_Z}{(\Lambda_1 + \Lambda_2) \left(1 + \sigma_Z^2 p_Z^2 + \frac{\sigma_\theta^2}{2}\right)} \right), \tag{30}$$

where p_Z is the solution of (26) given by

$$p_Z = \frac{\gamma \sigma_Z^2}{3(\Lambda_1 + \Lambda_2)} + \frac{2\gamma \sigma_\theta^2 (\gamma \sigma_Z \sigma_\theta)^{2/3}}{3(\Lambda_1 + \Lambda_2) \psi^{1/3}} + \frac{(\gamma \sigma_\theta)^{2/3} \psi^{1/3}}{6(\Lambda_1 + \Lambda_2) \sigma_Z^{2/3}}, \tag{31}$$

and

$$\psi = 8\gamma^2 \sigma_\theta^2 \sigma_Z^2 + 27(\Lambda_1 + \Lambda_2)^2 + 3\sqrt{3}(\Lambda_1 + \Lambda_2) \sqrt{16\gamma^2 \sigma_\theta^2 \sigma_Z^2 + 27(\Lambda_1 + \Lambda_2)^2}.$$

For convenience, we introduce $\widetilde{p}_Z = (\Lambda_1 + \Lambda_2) p_Z$. Note that $\widetilde{p}_Z(\Lambda_1 = 0, \Lambda_2 = 0) = \gamma \sigma_Z^2$. Thus, in the limit of a small fraction of informed traders, the expression (31) behaves almost as if there is no additional information propagation through different realizations of the signal.

The value of the individual signal is the decreasing continuously differentiable function of Λ_1. To show this,

$$\frac{d}{d\Lambda_1} \ln \left(1 + \frac{\gamma \sigma_Z^2 \widetilde{p}_Z}{2\Lambda_1^2 (2 + \sigma_\theta^2) + \sigma_Z^2 \widetilde{p}_Z^2} \right) =$$

$$- \frac{2\gamma \Lambda_1 \sigma_Z^2 (2 + \sigma_\theta^2) \left(2\widetilde{p}_Z - \left(\frac{\partial \widetilde{p}_Z}{\partial \Lambda_1}|_{\Lambda_2 = \Lambda_1}\right) \Lambda_1\right) + \gamma \sigma_Z^4 \widetilde{p}_Z \left(\frac{\partial \widetilde{p}_Z}{\partial \Lambda_1}|_{\Lambda_2 = \Lambda_1}\right)}{\left(2\Lambda_1^2 (2 + \sigma_\theta^2) + \sigma_Z^2 \widetilde{p}_Z^2 + \gamma \sigma_Z^2 \widetilde{p}_Z\right) \left(\widetilde{p}_Z^2 \sigma_Z^2 + 2\Lambda_1^2 (2 + \sigma_\theta^2)\right)}$$

and note that $\widetilde{p}_Z - \left(\frac{\partial \widetilde{p}_Z}{\partial \Lambda_1}|_{\Lambda_2=\Lambda_1}\right)\Lambda_1 > 0$. In addition, the value of the signal is convex for small values of Λ_1, and is becoming concave as Λ_1 increases. Thus, we can conclude that expression (30) has, at most, one maximum.

We can also calculate the expression for the second derivative of (30) at both small and large values of Λ_1. If the information providers serve just a small fraction of the market, $\Lambda_1 \ll 1$,

$$\frac{d^2\Pi_1}{d\Lambda_1^2}|_{\Lambda_2=\Lambda_1} = -\frac{5(4+3\sigma_\theta^2)\Lambda_1}{2\gamma^2\sigma_\theta^4\sigma_Z^2\left(1+\sigma_\theta^2\right)} + o(\Lambda_1^3),$$

and for the large, $\Lambda_1 \to \infty$,

$$\frac{d^2\Pi_1}{d\Lambda_1^2}|_{\Lambda_2=\Lambda_1} = -\frac{5}{18\Lambda_1\left(1+\sigma_\theta^2\right)}\left(\frac{\gamma\sigma_\theta\sigma_Z}{\Lambda_1}\right)^{4/3} + o(\Lambda_1^{-3}).$$

Thus, profit as a function of Λ_1 has $2k$ extremes, where $k = 0, 1, 2...$ We can conclude that the only option we have is $k = 0$, and problem (29) does not have an optimal solution. This result is not affected by the fact that we did not solve the problem for the optimal noise level, as it is valid for any σ_θ. ■

4. The effect of costly information gathering

In the Admati and Pfleiderer (1986) framework, the information provider is endowed with an information production technology that gives a signal with precision σ_θ^2 at zero cost. In real life, achieving certain precision of the signal comes at a cost that is increasing in precision. The mathematical form of the cost function should capture the fact that it is possible to obtain a signal that is perfectly precise, but only at infinite cost. At the same time, anyone can produce a signal with infinite variance at no cost at all,

$$\lim_{\sigma_\theta^2 \to 0} C_\theta = \infty, \lim_{\sigma_\theta^2 \to \infty} C_\theta = 0.$$

We shall consider the simplest case of such a function,

$$C_\theta = \begin{cases} 0, & \text{if } \sigma_\theta \geq \sigma_{\theta 0}, \\ \infty, & \text{if } \sigma_\theta < \sigma_{\theta 0}, \end{cases} \tag{32}$$

where $\sigma_{\theta 0}$ is a positive constant. It illustrates the notion that there exists some residual uncertainty that can never be captured by analysts.

First, we consider optimization in the C regime, where $\Lambda^{**} \geq 1$. It is easy to show that there exists, at most, one value of Λ that solves the unconstrained optimization problem of the information provider

$$\max_{0<\Lambda_i<\Lambda^{**},\sigma_\theta>\sigma_{\theta 0}} \Lambda_i \ln\left(1+\frac{2}{\sigma_\theta^2+4\left(\frac{1}{2}+\sigma_\theta^{-2}\right)\left(\frac{\Lambda_i}{\gamma\sigma_Z}\right)^2}\right). \tag{33}$$

This is done by showing that the second derivative of maximand in equation (33) changes sign only once, from negative to positive, and that the first derivative is positive for small Λ, and behaves as $-\Lambda^{-2}$ as Λ approaches infinity.

The condition (13) takes the form

$$\begin{aligned} \gamma^2\sigma_Z^2 &\geq 1+\sigma_\theta^{-2}+\sqrt{1+4\sigma_\theta^{-2}+5\sigma_\theta^{-4}}, \\ \sigma_\theta^2 &= \max\left(\frac{2}{\gamma\sigma_Z},\sigma_{\theta 0}^2\right). \end{aligned} \tag{34}$$

Obviously, if information providers were free to choose the optimal signal variance, the results would not change from those reported above. However, if the choice is constrained, the first order condition at Λ^{**} takes the form

$$\frac{4\left(1+\sigma_{\theta 0}^2-\sqrt{5+4\sigma_{\theta 0}^2+\sigma_{\theta 0}^4}\right)}{\left(2+\sigma_{\theta 0}^2\right)^2}+\ln\left(1+\frac{2\left(1+\sqrt{5+4\sigma_{\theta 0}^2+\sigma_{\theta 0}^4}\right)}{\left(2+\sigma_{\theta 0}^2\right)^2}\right). \tag{35}$$

The difference from the proof of Lemma 3 is very clear at this point. If the choice of noise variance is unconstrained, the analog of expression (35) is always positive, indicating that there is no $\Lambda \in [0, \Lambda^{**})$ satisfying the first order conditions. Conversely, in the constrained case, there exists a region of values, $\sigma_{\theta 0} < \overline{\sigma}_{\theta 0}$, where (35) is negative. Here $\overline{\sigma}_{\theta 0} \approx 0.675$ is the solution of first order conditions (35); it is easy to show that this solution exists, and is indeed unique. However, if the first order conditions are calculated at $\Lambda = 1$, it is clear that, for any $\gamma\sigma_Z > 1+\sqrt{2}$, it is of the same sign. Thus, the presence of the constraint on the choice of signal noise does not alter significantly the results of the previous section.

The boundaries between different regimes, however, are affected. From (34), $\gamma\sigma_Z$ becomes a decreasing function of $\sigma_{\theta 0}$. In the limit, as $\sigma_{\theta 0} \to \infty$, it would lead us to $\gamma\sigma_Z \geq \sqrt{2}$, replacing (34). We conclude that the presence of intrinsic limitations on the precision of the signal leads to an *increase* in the incidence of tacit collusion.

As the proof of Lemma 3 does not rely on optimization over signal noise, it is directly applicable in the constrained case as well. The results of this section can be summarized as follows:

Proposition 5. *Information provider-duopolists whose signals are uncorrelated do not limit the number of market participants. The charge for their information is*

$$\frac{1}{4\gamma}\ln\left(1+\frac{2}{\sigma_\theta^2+4\left(\frac{1}{2}+\sigma_\theta^{-2}\right)(\gamma\sigma_Z)^{-2}}\right) \tag{36}$$

where $\sigma_\theta^2=\max\left(\frac{2}{\gamma\sigma_Z},\sigma_{\theta 0}^2\right)$ *, and* $\sigma_{\theta 0}$ *is the highest precision available.*

It is interesting to note that the asset price has a maximum as a function of risk aversion γ.[5] The logic behind this is quite simple. If an agent is (almost) risk-neutral, his valuation of information about the variance of the risky asset returns is not very high. Clearly, for small γ, the value of information increases with risk aversion. In the opposite case, if risk aversion is high, the trader invests primarily in the risk-free asset, and valuation of information is decreasing in γ, as is the fraction of wealth invested in the risky asset.

The logic above is directly applicable to a market where the signals offered to traders are less than perfectly correlated, $\rho\in(0,1)$. The price of the signal is given by the analog of (36)

$$p=\Pi=\frac{1}{4\gamma}\ln\left(1+\frac{(1+\rho)^2\gamma^2\sigma_Z^2\sigma_\theta^2}{(1+\rho)^2\gamma^2\sigma_Z^2\sigma_\theta^4+4+2(1+\rho)\sigma_\theta^2}\right). \tag{37}$$

Similarly to the discussion above, if information providers can optimize over signal precision, they choose $\sigma_\theta^2=\max\left[\frac{2}{(1+\rho)\gamma\sigma_Z},\sigma_{\theta 0}^2\right]$, and price (profit) is

$$p=\Pi=\begin{cases}\frac{1}{4\gamma}\ln\left(1+\frac{\gamma^2\sigma_Z^2}{1+2\gamma\sigma_Z}\right), & \text{if } \sigma_{\theta 0}^2<\frac{2}{(1+\rho)\gamma\sigma_Z},\\ \frac{1}{4\gamma}\ln\left(1+\frac{\gamma^2\sigma_Z^2\sigma_{\theta 0}^2}{4+2\sigma_{\theta 0}^2+\gamma^2\sigma_Z^2\sigma_{\theta 0}^4}\right), & \text{if } \sigma_{\theta 0}^2\geq\frac{2}{(1+\rho)\gamma\sigma_Z}.\end{cases}$$

Note that the value of profit does not depend directly on the correlation between signals ρ. The choice of correlation affects only the choice of noise of the signal σ_θ. From the point of view of a trader who purchases both signals, the uncertainty of

[5] Numerical calculations shows that, in the unconstrained case, this maximum corresponds to $\gamma\sigma_Z\approx 1.5350$.

the variance of asset return is $(\gamma\sigma_Z)^{-1}$. Thus, if the information providers are free to choose optimal precision, the uncertainty of the agent's estimate does not depend on ρ. However, if the providers are constrained, the direct optimization of (37) over ρ shows that the optimum is reached at $\rho = 0$.