Liabilities—A new approach

An approach that avoids either asset only or full surplus optimization.

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Pension fund managers have shown faint interest in consideration of liabilities in pension fund asset allocation strategies. There are a variety of reasons for their lack of enthusiasm. First, the decision to abandon return maximization as a goal strikes ERISA-conditioned pension officers as imprudent, illegal, or not in their own personal interest. Moreover, suggested methods require an all-or-nothing approach to liability consideration — investors maximize either return or surplus; they are not offered a middle ground. Finally, in practice, surplus optimization or portfolio dedication normally requires acceptance of the idea that the accounting characterization of liabilities is, in fact, an accurate reflection of the true corporate liability.

This article presents a new approach for dealing with liabilities. Our method allows the effect of liability and asset comovement to be seen as a utility benefit for the portfolio, exactly analogous to, but in the opposite direction from, a risk penalty. In just the way that a return with unwanted volatility can be considered to offer the same utility as a lower certainty-equivalent return, a return with desired liability and asset comovement can be considered to offer as much utility as a higher return without that comovement.

We call the degree to which a particular asset or asset class can provide utility for an investor with a particular set of liabilities the *liability hedging credit* of the asset. The liability hedging credit is positively related to the covariance between the asset and the liability, positively related to the ratio of current assets to current liabilities, and inversely related to the risk tolerance of the investor.

The liability hedging credit can be calculated in a way that permits full or partial emphasis on liabilities. Full consideration yields the same result as surplus optimization, while zero consideration yields the same result as asset-only optimization. Partial consideration of liabilities allows the portfolio to take on hedging characteristics without the requirement of full surplus optimization.

Most importantly, the measure allows optimization to be performed at various levels of emphasis. The end result is a series of optimum portfolios with different levels of expected return, risk, and hedging. Knowledgeable pension managers can see precisely what they must give up in terms of one of these characteristics for a gain in one or both of the others.

The analytical underpinnings of liability hedging credits can be expanded to apply to other assets held by the investor but beyond the range of normal asset allocation decisions. Such extension allows portfolio managers to take into account the impact of the relationships between assets not subject to immediate allocation, such as real estate, venture capital and leveraged buy-outs, and the more traditional market-valued assets.

SURPLUS OPTIMIZATION

We start by assuming that the pension fund has determined what liabilities it wishes to consider in determining its investment strategy. This, in itself,

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is a difficult and confusing task, but not the subject under consideration here.

Let L represent the value of the relevant liability concept (for example, an "economic value" for the projected benefit obligation) and k the importance to be attached to it (e.g., 1.0 for a full surplus optimization). The relevant measure of surplus is

$$S = A - kL,$$

where A represents the value of the fund's assets and S is the surplus.

For example, if A = \$100, L = \$70, and k = 1,

$$S = \$100 - 1 \times \$70 = \$30,$$

which conforms to a traditional notion of surplus.

If A = \$100, L = \$70, and k = 0,

$$S = \$100 - 0 \times \$70 = \$100.$$

Here the "surplus" is equivalent to the fund's total assets, because zero emphasis is placed on the liabilities.

Today's surplus is known. Next year's is not. Letting subscripts 0 denote today and 1 next year, and tildes represent uncertain quantities:

$$S_0 = A_0 - kL_0, \text{ and}$$
$$\tilde{S}_1 = \tilde{A}_1 - k\tilde{L}_1.$$

A pension fund is concerned with next year's surplus. This amount can be expressed relative to today's asset value, i.e., $(\tilde{S}_1/A_0)$. This calculation is similar to the procedure followed when liabilities are ignored (i.e., k = 0). In such circumstances, the relevant concern is next year's asset value relative to today's. Relating future surplus to today's asset value is thus a natural extension of "asset-only" practice. More importantly, it maintains the dimensions in which risk tolerance is measured.

Because $\tilde{S}_1 = \tilde{A}_1 - k\tilde{L}_1$, the goal is to maximize

$$\frac{\tilde{A}_1}{A_0} - k\frac{\tilde{L}_1}{A_0}.$$

For reasons that will be made clear below, it is convenient to multiply the last term by (L_0/L_0) and rearrange terms to give

$$\frac{\tilde{A}_1}{A_0} - k\frac{L_0}{A_0}\frac{\tilde{L}_1}{L_0}.$$

This permits conversion to the more familiar notion of returns. Next year's asset value over today's equals $1 + \tilde{R}_A$, where $\tilde{R}_A$ is the rate of return on assets. Equivalently, next year's liability value over today's equals $1 + \tilde{R}_L$, where $\tilde{R}_L$ is the growth rate of the liabilities, which is defined here as the rate of return on liabilities. Note that these calculations exclude net cash inflows or outflows for the fund and net new accruals or paydowns for the liabilities.

Given these relationships, the equation can be rewritten as:

$$1 + \tilde{R}_A - k\frac{L_0}{A_0}(1 + \tilde{R}_L)$$

or

$$\left[1 - k\frac{L_0}{A_0}\right] + \left[\tilde{R}_A - k\frac{L_0}{A_0}\tilde{R}_L\right].$$

The first bracketed expression involves no uncertainty, so asset allocation decisions cannot affect it. For purposes of decision-making, one can concentrate entirely on the second expression. For convenience, let us denote this term $\tilde{Z}$:

$$\tilde{Z} \equiv \tilde{R}_A - k\frac{L_0}{A_0}\tilde{R}_L.$$

Let t be the fund's risk tolerance. Surplus optimization is designed to choose the asset mix that will maximize utility, defined as

$$U \equiv \text{Expected }(\tilde{Z}) - [\text{Variance }(\tilde{Z})/t],$$

taking the current mix, relevant transaction costs, and upper and lower bounds on holdings into account.

When k equals zero the variable of interest is $\tilde{R}_A$:

$$\tilde{Z} = \tilde{R}_A - 0 \times \frac{L_0}{A_0}\tilde{R}_L = \tilde{R}_A.$$

When k equals one,

$$\tilde{Z} = \tilde{R}_A - 1 \times \frac{L_0}{A_0}\tilde{R}_L = \tilde{R}_A - \frac{L_0}{A_0}\tilde{R}_L.$$

The importance of the liability return is directly related to the magnitude of the liability value vis-à-vis the asset value (L_0/A_0). Only if a plan has no surplus $(L_0/A_0 = 1)$ should one simply subtract the liability return from the asset return.

The better a plan's funding (i.e., the lower the value of (L_0/A_0), the smaller the effect of liabilities on asset allocation and the less the impact of a switch from an asset-only (k = 0) to a full surplus (k = 1) optimization.

Grouping terms somewhat differently:

$$\tilde{Z} = \tilde{R}_A - \left[k\frac{L_0}{A_0}\right]\tilde{R}_L.$$

The bracketed expression may be treated as the overall weight attached to the liability return. As can be seen, it will depend on the importance to be attached to liabilities (k) and their relative size (L_0/A_0).

LIABILITY HEDGING CREDITS

The goal of surplus optimization is to maximize utility, defined as

$$U \equiv \text{Expected}(\tilde{Z}) - [\text{Variance}(\tilde{Z})/t],$$

where

$$\tilde{Z} \equiv \tilde{R}_A - k\frac{L_0}{A_0}\tilde{R}_L.$$

The first term [Expected($\tilde{Z}$)] equals

$$\text{Expected}(\tilde{R}_A) - k\frac{L_0}{A_0}\text{Expected}(\tilde{R}_L).$$

Only the first term can be affected by asset allocation decisions. Thus only the expected return on assets needs to be considered — just as in a traditional asset allocation. The objective may thus be simplified to maximize:

$$\text{Expected}(\tilde{R}_A) - [\text{Variance}(\tilde{Z})/t]$$

without affecting the answer in any way.

This highlights an important point: Surplus optimization can be accomplished by changing only the risk that is considered, not the expected returns.

Standard formulas imply that the numerator of the second term, which represents risk, can be expressed as follows:

$$\text{Variance}(\tilde{Z}) = \text{Variance}(\tilde{R}_A) - 2k\frac{L_0}{A_0}$$

$$\text{Covariance}(\tilde{R}_A, \tilde{R}_L) + k^2\frac{L_0^2}{A_0^2}\text{Variance}(\tilde{R}_L),$$

where

Covariance ($\tilde{R}_A$, $\tilde{R}_L$) is the covariance between $\tilde{R}_A$ and $\tilde{R}_L$.

The last term includes only constants (the current liability and asset values and the importance attached to liabilities) and the risk associated with the future liability value. This term is unaffected by the asset allocation decisions and can be ignored without affecting the result.

This objective becomes:

$$\text{Maximize Expected}(\tilde{R}_A) - \left[\frac{\text{Variance}(\tilde{R}_A)}{t} - 2\frac{k}{t}\frac{L_0}{A_0}\text{Covariance}(\tilde{R}_A, \tilde{R}_L)\right],$$

or

$$\text{Maximize Expected}(\tilde{R}_A) - \frac{\text{Variance}(\tilde{R}_A)}{t} + 2\frac{k}{t}\frac{L_0}{A_0}\text{Covariance}(\tilde{R}_A, \tilde{R}_L).$$

The first two terms in these expressions are precisely equal to those employed for asset-only optimizations. The two together are often termed the risk-adjusted expected return — i.e., the expected return on assets less an appropriate risk penalty. Naming all three terms:

Utility = Expected Return − Risk Penalty
+ Liability Hedging Credit.

The risk penalty is equal to the variance (standard deviation squared) of the return on the assets divided by the investor's current risk tolerance. For example, if an asset mix has an expected return of 13% and a standard deviation of 10%, then for an investor with a (fairly typical) risk tolerance of 50:

Expected Return	13%	
Variance	$(10\%)^2$	= 100
Risk Penalty	100/50	= 2%

Risk-Adjusted
Expected Return 13% − 2% = 11%

For such an investor, this mix is as desirable as 11% for certain.

If the investor is concerned with liabilities (k > 0), the third term must be included to determine the overall desirability of an asset mix. This added term is called the liability hedging credit for the asset mix. Like the other two terms, it is stated in units of equivalent expected return per year. Thus an asset mix with a liability hedging credit of 1.5 would be equivalent to one offering 1.5% more in expected return but providing no liability hedging at all.

Extending the example above, assume that the asset mix in question has a liability hedging credit of 3%. Thus:

Expected Return	13%	
Variance	$(10\%)^2$	= 100
Risk Penalty	100/50	= 2%
Liability Hedging Credit	3%	

Risk-Adjusted
Expected Return 13% − 2% + 3% = 14%

For such an investor, this mix is as desirable as one offering 14% for certain but with no ability to serve as a hedge against fluctuations in liability values.

ASSET LIABILITY HEDGING CREDITS

Surplus optimization involves a willingness to accept lower expected return and/or greater asset risk in order to increase the ability of an asset mix to hedge against increases in liability values. Fund managers can find the best asset mix while taking liabilities into account to the desired extent by including the appropriate liability hedging credit in the calculation:

$$LHC_{mix} = \frac{2}{t}\frac{L_0}{A_0} \text{Covariance}(\tilde{R}_A, \tilde{R}_L),$$

where the subscript "mix" indicates that the value applies to the asset mix as a whole.

The greater the fund's tolerance for risk (t), the smaller the liability hedging credit. This conforms to intuition. The credit relates to the ability of an asset mix to reduce surplus risk. The greater one's tolerance for risk, the smaller the credit that should be awarded for reducing it. More importantly, the less the expected return that should be sacrificed to reduce such risk.

Conversely, the greater the value of the liabilities relative to assets, the greater the liability hedging credit. This also makes sense. The higher the relative magnitude of liabilities, the more credit should be awarded for a given percentage reduction in such risk. More importantly, the greater the expected return that should be sacrificed to achieve such a reduction.

The final term measures the covariance between the return on the asset mix and the return on the liabilities. If this equals zero, the return on the asset mix is uncorrelated with the return on the liabilities. Assets thus provide no hedging against liability increases, and the LHC is (appropriately) zero.

If, on the other hand, the covariance of $\tilde{R}_A$ with $\tilde{R}_L$ is positive, asset returns tend to be high when liability returns are high and low when liability returns are low. This provides some hedging against liability changes. The greater the covariance, the better this hedging ability, and the greater (appropriately) the LHC.

Most asset mixes have at least some tendency to hedge liability risks, but one could construct a mix with returns negatively correlated with liability returns. In such a case the LHC would be negative, indicating that the mix was worse than one with no hedging ability at all. Such a mix would tend to fall in value when liability values increase — exacerbating, not mitigating, the effect of the latter on the fund's surplus.

It is important to determine how the liability hedging credits for individual securities or asset classes combine to form the liability hedging credit for the portfolio mix. As the return on the asset mix is simply a weighted average of the returns on the assets, with the portfolio's relative market values as weights:

$$\tilde{R}_A = \sum_i X_i \tilde{R}_i,$$

where $\tilde{R}_i$, is the return on asset i, X_i is the proportion of the portfolio invested in asset i, and Σ denotes the summation of all such terms.

From this it follows that:

$$\text{Covariance}(\tilde{R}_A, \tilde{R}_L) = \sum_i X_i \text{Covariance}(\tilde{R}_i, \tilde{R}_L).$$

Thus the covariance of the return on the asset mix with the liabilities is simply the portfolio market-value-weighted average of the covariances of the component assets with the liabilities.

Finally, the liability hedging credit for any asset (i) can be defined as

$$LHC_i = \frac{2}{t}\frac{L_0}{A_0} \text{Covariance}(\tilde{R}_i, \tilde{R}_L).$$

The LHC of the entire mix will then be simply the portfolio market-value-weighted average of the LHCs of the asset classes:

$$LHC_{mix} = \sum_i X_i LHC_i.$$

COMPONENTS OF ASSET LIABILITY HEDGING CREDITS

The covariance of an asset with a liability is not particularly intuitive. Consideration of its components will permit further insight.

$$\text{Covariance}(\tilde{R}_i, \tilde{R}_L) = \rho_{iL}\sigma_i\sigma_L,$$

where

ρ_{iL} = the correlation between $\tilde{R}_i$ and $\tilde{R}_L$,

σ_i = the standard deviation (risk) of $\tilde{R}_i$, and

σ_L = the standard deviation (risk) of $\tilde{R}_L$.

Thus, other things equal, an asset class whose returns are highly correlated with liabilities will provide better liability hedging and receive a greater liability hedging credit; an asset with more risk will provide better liability hedging and receive a greater liability hedging credit; and the greater the liability risk, the greater all positive asset liability hedging credits.

Of course, other things are not always equal. For example, in the case of growth stocks and intermediate bonds, estimates of forward-looking risk and correlations relative to one plan's accumulated benefit obligation (ABO) liability were as in the Table.[1]

TABLE

Risk and Correlation Estimates

Asset Class	ρ_{iL}	σ_i	$\rho_{iL}\sigma_i$
Intermediate Bonds	0.87	4.20%	3.654%
Growth Stocks	0.21	19.75%	4.148%

In this case, growth stocks provided slightly more hedging ability than intermediate bonds. While their interest rate sensitivity was a smaller part of their total risk (witness the correlation coefficient of 0.21

versus 0.87), their total risk was much greater (19.75% versus 4.20%). The combined effect, obtained by multiplying the correlation coefficient by the standard deviation, is slightly greater for growth stocks. Here, intermediate bonds offered no advantage vis-à-vis growth stocks in terms of liability hedging.

USE OF LIABILITY HEDGING CREDITS

Our derivations show that those concerned with liabilities need add only one new component to measures they traditionally use for asset allocation: a liability hedging credit. The LHC for any asset mix equals a market-value-weighted average of the liability hedging credits of the assets:

$$LHC_{mix} = \sum_i X_i LHC_i.$$

The liability hedging credit for an asset will depend on the concept of liabilities chosen and the:

1. Fund's risk tolerance (t);
2. Importance to be attached to liabilities (k);
3. Current value of liabilities relative to assets (L_0/A_0);
4. Correlation of the asset's return with the liabilities (ρ_{iL});
5. Asset's risk (σ_i); and
6. Liability risk (σ_L).

More precisely,

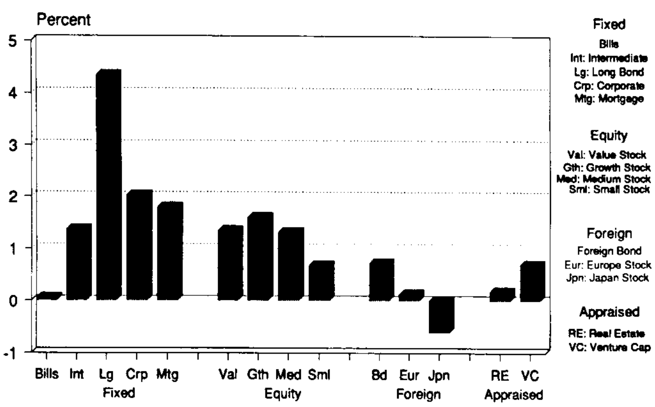
$$LHC_i = \frac{2}{t} k \frac{L_0}{A_0} \rho_{iL}\sigma_i\sigma_L.$$

The Figure shows liability hedging credits for one fund's projected benefit obligation (PBO) at the end of March 1989, with full emphasis placed on liabilities (k = 1).[2] Note that long government bonds provide by far the best hedging ability and Japanese stocks the worst. Note also that some of the differences are quite large. For example, long government bonds provide an LHC of 4.3% compared to 1.5% for growth stocks — a difference of 2.8%.

To interpret these numbers, it is important to understand that the optimal asset mix is the same that would be obtained if each of these values were added directly to the expected return for the asset in question, and a traditional mean/variance asset allocation optimization were then performed. This suggests how the liability hedging credits should be used. By adding them to the expected returns, the manager of the fund gives long bonds a "bonus" equivalent to 2.8% per year over growth stocks. This is appropriate if changes in PBO surplus really deserve full emphasis.

FIGURE 1

LIABILITY HEDGING CREDITS: PBO
MARCH 1989



OTHER APPLICATIONS

We have focused so far on liability hedging. This technique for dealing with liabilities also can be applied to situations where liabilities are not necessarily involved, but where other assets beyond the asset allocator's decisions form part of the beneficial owner's net worth. An individual investor, for example, might wish to consider his or her house, social security benefits, and various other assets when determining the optimal asset allocation among securities.

The value of next period's "surplus" (net worth) in this case may be written:

$$\tilde{S}_1 = \tilde{A}_1 + k\tilde{O}_1$$

where $\tilde{O}_1$ represents the (uncertain) future value of the "other assets," and k represents the emphasis to be placed on these assets. As before, dividing by current asset value provides an objective that conforms to standard usage, giving:

$$\frac{\tilde{A}_1}{A_0} + k \frac{O_0}{A_0}\frac{\tilde{O}_1}{O_0}.$$

The derivation need not be continued beyond this point, as the equation differs from that for liabilities in only one respect: the terms are related by a plus sign rather than a minus sign. This makes sense, because an asset can be considered a negative liability. A "negative hedging credit" can be obtained simply by reversing the signs and altering the notation appropriately. Thus, from:

$$LHC_i = \frac{2}{t} k \frac{L_0}{A_0} \rho_{iL}\sigma_i\sigma_L$$

one can obtain:

$$OACP_i = \frac{2}{t} k \frac{O_0}{A_0} \rho_{iO}\sigma_i\sigma_O,$$

where $OACP_i$ represents the "Other Asset Covariance Penalty" for asset i and is to be subtracted from expected return when determining the asset's contribution to overall utility. Other things equal, the greater an asset's covariance with the "other assets," the less desirable it is.

Some cases will involve both "other assets" and liabilities, and some may involve multiple types of "other assets" and/or liabilities. It might seem that all such considerations must be grouped together and analyzed as a single entity, but this is not necessary. Rather, each can be analyzed in the manner shown above, and the relevant liability hedging credits and/or other asset covariance penalties simply added together.

We can show this in the case of a fund with other assets (O) and liabilities (L). For simplicity, it is assumed that full emphasis is to be placed on each.

In this case, next period's "surplus" (net worth) is:

$$\tilde{S}_1 = \tilde{A}_1 + \tilde{O}_1 - \tilde{L}_1.$$

Dividing by current assets gives:

$$\frac{\tilde{A}_1}{A_0} + \frac{O_0}{A_0}\frac{\tilde{O}_1}{O_0} - \frac{L_0}{A_0}\frac{\tilde{L}_1}{L_0}.$$

Note that only the first term can be affected by the asset allocation decision. To see the essence of the analysis, the three terms can be represented as:

$$\tilde{Z} = \tilde{a} + \tilde{b} + \tilde{c},$$

where the signs are included in the definitions of the new variables, and only $\tilde{a}$ can be affected by the asset allocation decision.

As always, the objective is to maximize:

$$U = \text{Expected}\ (\tilde{Z}) - [\text{Variance}(\tilde{Z})/t].$$

The expected value will be:

$$\text{Expected}\ (\tilde{a}) + \text{Expected}\ (\tilde{b}) + \text{Expected}\ (\tilde{c}).$$

Because the latter two terms are unaffected by asset allocation decisions, they may be dropped from the objective function without affecting the outcome. This leaves:

$$U = \text{Expected}\ (\tilde{a}) - [\text{Variance}\ (\tilde{Z})/t].$$

As before, only the expected return on the assets matters.

The variance of Z will have several terms:

$$\text{Variance}\ (\tilde{a}) + \text{Variance}\ (\tilde{b}) + \text{Variance}\ (\tilde{c}) + 2\text{Covariance}\ (\tilde{a},\tilde{b}) + 2\text{Covariance}\ (\tilde{a},\tilde{c}) + 2\text{Covariance}\ (\tilde{b},\tilde{c}).$$

Only three are affected by the asset allocation decision. Their sum is:

$$\text{Variance}\ (\tilde{a}) + 2\text{Covariance}\ (\tilde{a},\tilde{b}) + 2\text{Covariance}\ (\tilde{a},\tilde{c}).$$

The objective function can thus be written as:

$$\text{Maximize Expected}\ (\tilde{a}) - [\text{Variance}\ (\tilde{a})/t] - [2\text{Covariance}\ (\tilde{a},\tilde{b})/t] - [2\text{Covariance}\ (\tilde{a},\tilde{c})/t]$$

The first two terms constitute the usual measure of utility in an "asset only" context. The third and fourth correspond to the type of "other asset covariance penalty" or "liability hedging credit" derived earlier.

These principles are general. The only effects of other assets or liabilities on asset allocation arise as a result of covariances between the assets being allocated and the "outside" assets or liabilities. As covariances are additive, each such influence may be analyzed separately, and the results added to (or subtracted from) the asset's expected returns prior to performing the asset allocation analysis. The result will be an asset mix that provides the appropriate amount of utility, based on the chosen concept of the net worth of the beneficial owner.

SUMMARY

This article provides a procedure allowing proper incorporation of any coexisting assets or liabilities into the asset mix considerations for the remainder of a portfolio. The end result is an asset mix that is developed through a traditional mean/variance asset-only optimization but that maximizes utility for the beneficial owner of the portfolio while taking these other factors into account.

The liability hedging credits and other asset covariance penalties that form the basis for the procedure are calculated in a straightforward manner. They can be applied in a way that permits full or partial consideration of the movements of the other assets or liabilities. They can be developed separately and are additive across these other assets or liabilities. Once calculated, they have merely to be added to the expected returns on the assets to provide the proper values for the optimization.

This procedure permits the pension officer to avoid either asset-only or full surplus optimization. It allows for sensitivity analysis to determine the effects of greater or lesser emphasis on other assets or liabilities. It permits the use and development of liability descriptions that do not necessarily conform to traditional accounting and actuarial standards. Finally, it can be used to measure the exact relationships among expected returns, risks, and hedging characteristics.

Pension plan asset allocators can see exactly what must be sacrificed in terms of one or two of these characteristics to improve the third. The procedure allows them to make the appropriate investment choices for the asset classes under their control, yet take into consideration the impacts of others. This should lead, in the long run, to a fund optimally tailored for its intended purposes.

[1] The values of the estimated forward-looking standard deviations and correlations used in the table were developed through the use of the proprietary risk and liability estimation procedures of Sharpe–Tint, Inc.

[2] The figure is developed using the asset and liability values of a large defined-benefit pension plan and Sharpe-Tint, Inc.'s, estimates of that plan's risk tolerance and forward-looking asset class standard deviations and correlations.